10. [7 points] Consider the continuous function

$$f(x) = \begin{cases} -2 - \ln(x+2) & -2 < x \le -1 \\ x2^{-x} & x > -1 \end{cases}$$

and its derivative

$$f'(x) = \begin{cases} -\dfrac{1}{x+2} & -2 < x < -1 \\ 2^{-x}(1 - x\ln(2)) & x > -1. \end{cases}$$

a. [2 points] Find all critical point(s) of $f(x)$. Write NONE if there are none.

Solution: We need to look for values of x where $f'(x)$ is zero or undefined. We can see that the first piece of $f'(x)$ is undefined at -2, but that's not in the domain, so it is not a critical point. We can also solve to see that the second piece of $f'(x)$ is zero when $x = \frac{1}{\ln(2)} \approx 1.443$, so this is in the domain and so a critical point.

We also can't forget that $f'(-1)$ may not be defined because there is where the pieces meet. In fact, the piecewise function given for $f'(x)$ isn't defined at $x = -1$. We can also check this by seeing that the two pieces of $f'(x)$ are not equal when we plug in $x = -1$. (The first piece is -1, while the second is $2(1 + \ln(2)) > 0$, so $f(x)$ has a corner here.)

Answer: critical point(s) at $x =$ $-1,\ \dfrac{1}{\ln(2)}$

b. [5 points] Find the x-coordinate of all global maxima and global minima of $f(x)$ on its domain $(-2, \infty)$. For each, write NONE if there are none. You must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers.

Solution: The critical points of $f(x)$ are $x = -1$, and $1/\ln(2)$ as we found above. We are optimizing on the domain $(-2, \infty)$, so aside from the critical points, we also need to consider the behavior at the endpoints $x = -2$ and the limit as x goes to ∞.

$\lim_{x\to -2^+} f(x)$	$+\infty$
$f(-1)$	-2
$f(1/\ln(2))$	≈ 0.53074
$\lim_{x\to\infty} f(x)$	0

The smallest output value is -2, so $x = -1$ is the global minimum. Since $\lim_{x\to -2^+} f(x) = +\infty$, there is no global maximum

Answer: global max(es) at $x =$ **NONE**

Answer: global min(s) at $x =$ -1