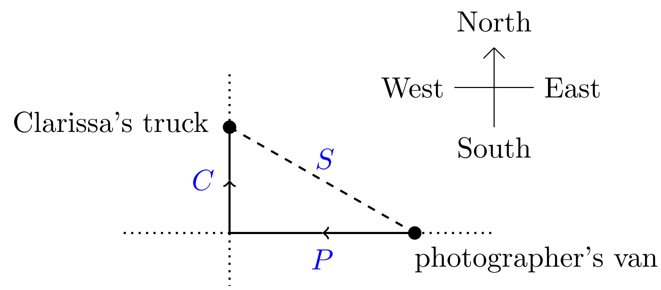
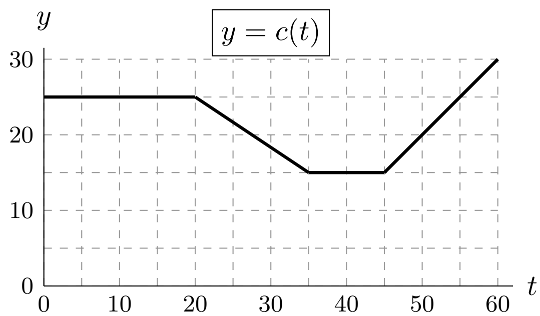
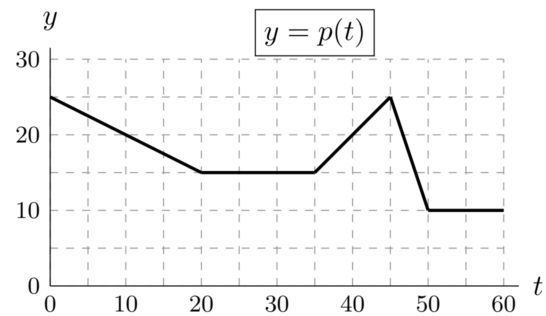
7. [10 points] Clarissa is driving a truck of showpigs, heading due north away from an intersection, while her photographer's van is due east of the intersection and driving due west, as shown to the right.



For time t measured in seconds after noon, let $c(t)$ be the speed of Clarissa's truck, and let $p(t)$ be the speed of the photographer's van, both measured in meters per second (m/s). Graphs of $c(t)$ and $p(t)$ are shown below.





At $t = 20$, Clarissa is 1000 meters from the intersection, and her photographer's van is 2400 meters from the intersection. Throughout this problem, be sure your work is clear.

a. [5 points] At $t = 20$, is the distance between the two vehicles increasing or decreasing? At what rate?

Solution: We can label the triangle as in the above picture. By the Pythagorean Theorem, $C^2 + P^2 = S^2$. Taking the derivative of both sides, we see that

$$2C\frac{dC}{dt} + 2P\frac{dP}{dt} = 2S\frac{dS}{dt}.$$

At time $t = 20$, we have $C = 1000$, $P = 2400$, and $S = \sqrt{C^2 + P^2} = 2600$. Since Clarissa is moving *away* from the intersection, $\frac{dC}{dt} = c(20) = 25$ and since the photographer is moving *towards* it, $\frac{dP}{dt} = -p(20) = -15$. Plugging in this information, we find that $\frac{dS}{dt} \approx -4.23077$.

Answer: INCREASING **[DECREASING]** at a rate of: ≈ 4.23077 m/s

b. [2 points] How far from the intersection is the photographer's van at $t = 35$?

Solution:

$$P(35) = 2400 - \int_{20}^{35} p(t)dt = 2400 - 225 = 2175$$

Answer: 2175 meters

c. [3 points] What is the distance, in meters, between the two vehicles at $t = 35$?

Solution:

$$C(35) = 1000 + \int_{20}^{35} c(t)dt = 1000 + 300 = 1300$$

so using our answer from part (b), $S(35) = \sqrt{(2175)^2 + (1300)^2} \approx 2533.90$.

Answer: 2533.9 meters