

8. [9 points] Given below is a table of values for a function $g(x)$ and its derivative $g'(x)$. The functions $g(x)$, $g'(x)$, and $g''(x)$ are all defined and continuous for all real numbers.

x	-3	-2	0	2	3	4	6	8
$g(x)$	2	3	7	9	5	1	-5	-7
$g'(x)$	0	4	1	0	-2	-4	-1	-3

Assume that between consecutive values of x given in the table above, $g(x)$ is either **always increasing** or **always decreasing**.

Find the quantities in **a.–c.** exactly, or write NEI if there is not enough information provided to do so. You do not need to show work, but limited partial credit may be awarded for work shown.

a. [1 point] $\int_3^6 g(x) dx$

Answer: NEI

b. [2 points] $\int_{-2}^2 3g'(x) dx$

Solution: $3(g(2) - g(-2)) = 3(9 - 3) = 18$

Answer: 18

c. [3 points] $\int_0^4 (g''(x) + x) dx$

Solution:

$$\begin{aligned} \int_0^4 (g''(x) + x) dx &= \int_0^4 g''(x) dx + \int_0^4 x dx = (g'(4) - g'(0)) + \left(\frac{(4)^2}{2} - \frac{(0)^2}{2} \right) \\ &= (-4 - 1) + 8 \end{aligned}$$

Answer: 3

- d. [2 points] Use a right-hand Riemann sum with three equal subdivisions to estimate $\int_2^8 g(x) dx$.

Write out all the terms in your sum.

Solution: $\Delta x = (8 - 2)/3 = 2$, so the Riemann sum is

$$g(4) \cdot 2 + g(6) \cdot 2 + g(8) \cdot 2 = 2(1 + (-5) + (-7)) = -22$$

- e. [1 point] Does the answer to part **d.** overestimate, underestimate, or equal the value of $\int_2^8 g(x) dx$? Circle your answer. If there is not enough information, circle NEI.

Answer: OVERESTIMATE UNDERESTIMATE EQUAL NEI