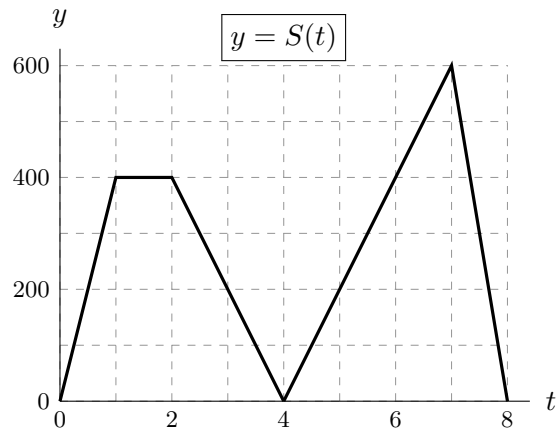
9. [9 points] Students from two rival universities had a competition to see who could clean up the most litter at a nature preserve.

University A went first, cleaning up litter from noon to 4pm. Each student from University A cleaned at a rate of 12 pounds of litter per hour.

Then University B cleaned up litter from 4pm to 8pm. Each student from University B cleaned at a rate of 9 pounds of litter per hour.

Let $S(t)$ be the number of students cleaning up litter at time t hours past noon. The graph of $S(t)$ is shown to the right.



a. [2 points] Find the total amount of litter cleaned up by University A. Show your work.

Solution:

$$12\int_0^4 S(t)\ dt = 12(1000)$$

Answer: 12000 pounds

b. [3 points] Find the total amount of litter cleaned up throughout the entire eight-hour competition. Show your work.

Solution:

$$12\int_0^4 S(t)dt + 9\int_4^8 S(t)\ dt = 12(1000) + 9(1200)$$

Answer: 22800 pounds

c. [4 points] The competition was broadcast live on TV. The number of people viewing the TV broadcast at time t hours past noon is given by the function

$$B(t) = 4S(t) + 200.$$

Find the average number of people viewing TV broadcast during the eight-hour competition.

Solution:

$$\begin{aligned}\frac{1}{8}\int_0^8 (4P(t) + 200)\ dt &= \frac{4}{8}\int_0^8 S(t)\ dt + \frac{1}{8}\int_0^8 200\ dt \\ &= \frac{1}{2}(1000 + 1200) + \frac{1}{8}(200 \cdot 8)\end{aligned}$$

Answer: 1300 people