

1. [13 points] Given below is a table of values for a function  $h(x)$  and its derivative  $h'(x)$ . The functions  $h(x)$ ,  $h'(x)$ ,  $h''(x)$ , and  $h'''(x)$  are all defined and continuous on  $(-\infty, \infty)$ .

$x$	-6	-4	-2	0	2	4	6
$h(x)$	2	-0.5	-2	-3	1	4	3
$h'(x)$	0	-4	-1	0	3	0	-2

Assume that between consecutive values of  $x$  given in the table above,  $h(x)$  is either **always increasing** or **always decreasing**.

In **a.–c.**, find the numerical value **exactly**, or write NEI if there is not enough information provided to do so. *You do not need to show work on this page, but limited partial credit may be awarded for work shown.*

- a. [2 points] Find the average rate of change of  $h(x)$  from  $x = -6$  to  $x = -2$ .

**Answer:**  $\frac{-2 - 2}{-2 - (-6)} = -1$

- b. [2 points] If the average value of  $h'''(x)$  on the interval  $[-6, 0]$  is 2, find  $5 \cdot \int_{-6}^0 (1 + h'''(x)) dx$ .

*Solution:*  $\frac{1}{6} \int_{-6}^0 h'''(x) dx = 2$ , so  $\int_{-6}^0 h'''(x) dx = 12$ . Then  $5 \cdot \int_{-6}^0 (1 + h'''(x)) dx = 5 \cdot 6 + 5 \cdot 12$

**Answer:**  $90$

- c. [3 points] Find  $\int_{-4}^{-2} (2h'(x) + x) dx$ .

*Solution:*  $= 2 \int_{-4}^{-2} h'(x) dx + \int_{-4}^{-2} x dx = 2(h(-2) - h(4)) + \frac{x^2}{2} \Big|_{-4}^{-2} = 2(-2 - (-0.5)) + \frac{4}{2} - \frac{16}{2}$

**Answer:**  $-9$

- d. [2 points] Find an equation for the tangent line to the graph of  $h(x)$  at  $x = 6$ .

**Answer:**  $y - 3 = -2(x - 6)$  or  $y = -2x + 15$

Given below is a table of values for a function  $h(x)$  and its derivative  $h'(x)$ . The functions  $h(x)$ ,  $h'(x)$ ,  $h''(x)$ , and  $h'''(x)$  are all defined and continuous on  $(-\infty, \infty)$ .

$x$	-6	-4	-2	0	2	4	6
$h(x)$	2	-0.5	-2	-3	1	4	3
$h'(x)$	0	-4	-1	0	3	0	-2

Assume that between consecutive values of  $x$  given in the table above,  $h(x)$  is either **always increasing** or **always decreasing**.

- e. [2 points] Use a left Riemann sum with three equal subdivisions to estimate  $\int_{-6}^6 h(x) dx$ . Write out all the terms in your sum, which you do not need to simplify.

*Solution:*

$$\begin{aligned} & 4 \cdot h(-6) + 4 \cdot h(-2) + 4 \cdot h(2) \\ &= 4 \cdot 2 + 4 \cdot (-2) + 4 \cdot (1) \end{aligned}$$

- f. [2 points] Fill in each blank below with one of the following:

$$\boxed{\leq}, \quad \boxed{\geq}, \quad \boxed{=} \text{ or } \boxed{\text{NEI}}$$

where NEI means there is not enough information to decide. You need not justify your answers.

i.  $\int_{-6}^0 h(x) dx$       $\leq$       $2h(-6) + 2h(-4) + 2h(-2)$ .

ii.  $\int_0^6 h(x) dx$      NEI      $2h(0) + 2h(2) + 2h(4)$ .