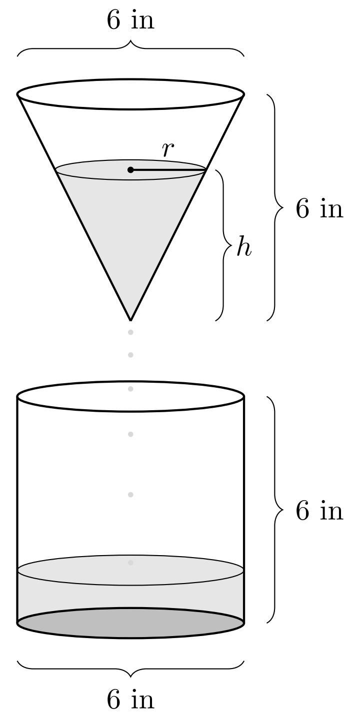


**3**. [9 points] Coffee is draining from a cone filter into a cylindrical pot, as shown in the figure to the right. The height and diameter of both the filter and the coffee pot are 6 inches.

Let $r$ be the radius of the circular surface and $h$ be the height of the coffee remaining in the filter, both measured in inches. Note that the shape of the filter implies that $r = \dfrac{h}{2}$.

*Recall that the volume of a cone with radius r and height h is* $\frac{1}{3}\pi r^2 h$*, while the volume of a circular cylinder with radius R and height H is* $\pi R^2 H$.

At the moment in time when the height of the coffee in the filter is 5 inches, the coffee is draining from the filter at a rate of 10 cubic inches per minute.

**a**. [5 points] At what rate is the height of the coffee in the filter decreasing at that moment? *Include units.*

*Solution:* Let $V$ be the volume of coffee in the filter, so

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3.$$

Differentiating with respect to $t$, we get

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}.$$

Since $\frac{dV}{dt} = -10$ and $h = 5$, this means

$$-10 = \frac{1}{4}\pi(5)^2\frac{dh}{dt}, \quad \text{so} \quad \frac{dh}{dt} = \frac{-40}{25\pi} = \frac{-8}{5\pi}.$$

Thus the height of the coffee is decreasing at a rate of $\frac{8}{5\pi}$ inches per minute.

**Answer:** $\frac{8}{5\pi}$ **inches per minute**

**b**. [4 points] At what rate is the height of the coffee in the pot increasing at that moment? *Include units.*

*Solution:* Let $V$ be the volume and $H$ the height of coffee in the pot, so

$$V = \pi \cdot 3^2 H = 9\pi H.$$

Differentiating with respect to $t$, we get

$$\frac{dV}{dt} = 9\pi \frac{dH}{dt},$$

so

$$\frac{dH}{dt} = \frac{10}{9\pi}.$$

Thus the height of coffee in the pot is increasing at a rate of $\frac{10}{9\pi}$ inches per minute.

**Answer:** $\frac{10}{9\pi}$ **inches per minute**