7. [9 points] Consider the family of functions

$$m(x) = x + \frac{c^2}{x}$$

defined for $x > 0$, where c is a positive constant.

Throughout this problem, use calculus to find your answers, show all your work, and be sure to show enough evidence to justify your conclusions.

a. [2 points] Any function in this family has only one critical point on its domain $x > 0$. In terms of c, what is the x-coordinate of this critical point?

Solution: $m'(x) = 1 - \frac{c^2}{x^2}$.
$m'(x)$ DNE at $x = 0$ which is **not** in the domain.
$m'(x) = 0$ at $x = c$ which **is** in the domain.

Answer: c **or** $x = c$

b. [3 points] Is the critical point a local minimum, a local maximum, neither, or is there not enough information to decide? Circle your answer below.

Solution:
For the 1^{st} *derivative test:* $m'(\frac{c}{2}) = 1 - 4\frac{c^2}{c^2} = -3 < 0$ and $m'(2c) = 1 - \frac{c^2}{4c^2} = \frac{3}{4} > 0$. Therefore $x = c$ is a **local min**.
For the 2^{nd} *derivative test:* $m''(x) = 0 + 2\frac{c^2}{x^3}$ and $m''(c) = \frac{2}{c} > 0$. Therefore $x = c$ is a **local min**.

Answer: [local min] local max neither not enough info

c. [2 points] Find the x-coordinates of all inflection points of $m(x)$, or if there are none, write NONE.

Solution: $m''(x) = 0 + 2\frac{c^2}{x^3}$ which is defined everywhere in the domain and not equal to zero on the domain. Therefore $m(x)$ has no inflection points.

Answer: Inflection point(s) at $x =$ **NONE**

d. [2 points] Find the value for c such that $m(x) = 10$ at its critical point.

Solution: The value at $x = c$ is $m(c) = c + \frac{c^2}{c} = 2c$. If it is equal to 10, then $2c = 10$, or $c = 5$.

Answer: $c =$ 5