8. [10 points] Scientists are continuing their study of water temperature in Lake Michigan using their underwater drone. Assume the following functions $W(p)$ and $T(p)$ are invertible and differentiable:

- Let $W(p)$ be the temperature, in degrees Celsius (°C), of the water at a depth of p feet.
- Let $T(p)$ be the time, in minutes, that it takes for the drone to descend to a depth of p feet.

a. [2 points] Write a single equation representing the following statement in terms of the functions W, T, and/or their inverses:

It takes the drone 3 minutes to reach water with a temperature of 5.7°C.

Answer: $T\left(W^{-1}(5.7)\right) = 3$ **or** $W\left(T^{-1}(3)\right) = 5.7$

b. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$W'(50) = -0.2.$$

Compared to the water at a depth of 50 feet, the water at a depth of 53 feet...

Solution: ...is about 0.6°C colder.

c. [3 points] Use a complete sentence to give a practical interpretation of the equation

$$\int_{50}^{65} T'(p)\ dp = 1.$$

Solution: It takes the drone one minute to descend from a depth of 50 ft to a depth of 65 ft.

d. [2 points] Which of the following expressions gives the average temperature, in °C, of the water outside of the drone during the first 5 minutes of its descent? Circle the one correct answer.

i. $\dfrac{1}{5}\displaystyle\int_0^5 W(p)\ dp$

ii. $\dfrac{1}{5}\displaystyle\int_0^5 W(T^{-1}(t))\ dt$

iii. $\dfrac{1}{5}\displaystyle\int_0^5 W'(T(t)) \cdot T'(t)\ dt$

iv. $\dfrac{W(T^{-1}(5)) - W(T^{-1}(0))}{5}$

v. $\dfrac{W(T(5)) + W(T(0))}{2}$

vi. $\dfrac{T(W^{-1}(5)) - T(W^{-1}(0))}{5}$