4. [6 points] The expressions below define five different functions of \( x \), labeled i. – v., which are used for answers in parts a. and b.

\[
\begin{align*}
i. & \quad \frac{x^2 + 1}{x - 3} \\
ii. & \quad \frac{x^2 - 1}{x^3 - 8} \\
iii. & \quad \frac{x}{e^x} \\
iv. & \quad \frac{e^x}{x} \\
v. & \quad \frac{(x - 2)^{1/3}}{x}
\end{align*}
\]

a. [2 points] Which of the above functions have a limit of 0 as \( x \to \infty \)? Circle all correct answers.

i. ii. iii. iv. v. none of these

b. [2 points] Which of the above functions satisfy the hypotheses of the Mean Value Theorem on the interval [1, 3]? Circle all correct answers.

i. ii. iii. iv. v. none of these

c. [2 points] Circle the one correct equality below to complete the statement of the Mean Value Theorem applied to the function \( f(x) = e^x/x \) on the interval [1, 3]:

“There is a point \( c \) in the interval (1, 3) such that . . .”

\[
\frac{xe^x - e^x}{x^2} = \frac{1}{2} \left( \frac{e^c}{c} - e \right) \quad \frac{ce^c - e^c}{c^2} = \frac{e^3}{9} \quad \frac{e^c}{c} = \frac{1}{2} \left( \frac{e^3 - e}{3 - 1} \right) \quad \frac{e^c(c - 1)}{c^2} = \frac{e^3 - e}{6} - \frac{c}{2}
\]

5. [5 points] Find the global maximum and minimum output values of the function

\[ y = w(x) = 3 - 2x^2 + \frac{x^4}{2} \]

on the interval \([-1, 2]\), and list all \( x \)-values at which the global maximum and minimum occur. Show all your work, and use calculus to justify your answers, which should be numerical.

Answer: On the interval \([-1, 2]\), the function \( w(x) \) has a . . .

   global maximum value of \( y = \ldots \), occurring at the point(s) \( x = \ldots \),

   and a global minimum value of \( y = \ldots \), occurring at the point(s) \( x = \ldots \).