7. [ 9 points] In an unexpected twist, Carson Soltonni also runs a business selling vacuum cleaners out of his house. The cost in hundreds of dollars for him to produce $q$ hundred vacuum cleaners is

$$
C(q)=\frac{q^{3}}{3}-5 q^{2}+59 q+5 .
$$

Carson sells his vacuum cleaners for 50 dollars each, and he is trying to determine how many to sell in order to maximize profit. Some values of $C(q)$, rounded to the nearest integer, are given in the table below.

| $q$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C(q)$ | 59 | 106 | 146 | 182 | 217 | 251 | 287 | 328 | 374 |

a. [1 point] What is the fixed cost of Carson's business?

## Answer:

$\qquad$ hundred dollars.
b. [3 points] Find the marginal revenue function $M R(q)$ and marginal cost function $M C(q)$ of Carson's business, in hundreds of dollars per hundred vacuum cleaners.

Answer: $\quad M R(q)=$ $\qquad$ and $M C(q)=$ $\qquad$
c. [3 points] How many vacuum cleaners should Carson produce and sell to maximize profit? Show your work and use calculus. You do not need to fully justify your answer, but partial credit may be awarded for work shown.

Answer: $\qquad$ hundred vacuum cleaners.
d. [2 points] Unsure how to solve the calculus problem in part c., Carson just decides to produce and sell as many vacuum cleaners as he can. Unfortunately, a court order terminates Carson's business immediately after he had produced and sold 600 vacuum cleaners. At this point, had Carson's business gained or lost money? How much?
Give your answer by circling GAINED or LOST and writing a positive number on the blank.
$\qquad$

