8. [11 points] Consider the family of functions $f(x) = x^4 e^{-cx}$, where $c > 0$. Note that

$$f'(x) = x^3 e^{-cx}(4 - cx) \qquad \text{and} \qquad f''(x) = c^2 x^2 e^{-cx}\left(x - \tfrac{2}{c}\right)\left(x - \tfrac{6}{c}\right).$$

a. [2 points] List all the critical points of $f(x)$ and $f'(x)$, in terms of c. *No justification necessary.*

Answer: Critical points of $f(x)$: ______________. Critical points of $f'(x)$: ______________.

b. [4 points] Determine, in terms of the parameter c, the intervals of concavity of the function $f(x)$. Use calculus to find your answers, show all your work, and be sure to show enough evidence to justify your conclusions.

Answer: Intervals on which $f(x)$ is concave up: ______________________________

Answer: Intervals on which $f(x)$ is concave down: ______________________________

c. [2 points] Circle the x-coordinates of all inflection points of $f(x)$ in terms of the parameter c that are listed below, or, if no inflection points of $f(x)$ are listed below, circle NONE.

0 $\qquad e^{-c}$ $\qquad e^{c}$ $\qquad \dfrac{2}{c}$ $\qquad \dfrac{4}{c}$ $\qquad \dfrac{6}{c}$ $\qquad$ NONE

d. [3 points] For each $c > 0$, the function $y = f(x)$ has exactly one local extremum in $(0, \infty)$.

i. Find the unique value of c such that $f(x)$ has a local extreme value of $y = 1$ in the interval $(0, \infty)$. *Show your work.*

Answer: $c =$ ______________

ii. Is the local extremum of $f(x)$ in $(0, \infty)$ a max or a min? Circle your answer: MAX MIN *(No justification is necessary.)*