1. [14 points] Given below is a table of some values of the **even** function $k(x)$, along with its derivative $k'(x)$. Assume the functions $k(x)$, $k'(x)$, and $k''(x)$ are continuous on $(-\infty, \infty)$, and that $k(x)$ is decreasing on $(0, \infty)$. Your final answers in this problem should not include the letter k.

x	0	1	2	3	4	5
$k(x)$	12	8	7	2	0	-3
$k'(x)$	0	-3	-4	-2	-1	-5

In parts **a.–c.**, find the numerical value **exactly**, or write NEI if there is not enough information provided to do so. *Show your work. Limited partial credit may be awarded for work shown.*

a. [2 points] Find $\displaystyle\int_1^3 \left(2k'(x) + e^x\right) dx$.

Solution:

$$\int_1^3 \left(2k'(x)+e^x\right) dx \;=\; 2\int_1^3 k'(x)\,dx + \int_1^3 e^x\,dx \;=\; 2\big(k(3)-k(1)\big)+\left(e^3-e^1\right) \;=\; 2(2-8)+e^3-e.$$

Answer: $-12 + e^3 - e$

b. [2 points] Find the average value of $k''(x)$ on the interval $[1, 4]$.

Solution: The average value of $k''(x)$ on $[1,4]$ is

$$\frac{1}{4-1}\int_1^4 k''(x)\,dx \;=\; \frac{1}{3}\big(k'(4)-k'(1)\big) \;=\; \frac{-1-(-3)}{3} \;=\; \frac{2}{3}.$$

Answer: $2/3$

c. [2 points] Find $\displaystyle\lim_{h\to 0} k(-1) + \lim_{h\to 0} \frac{k(3+h)-k(3)}{h}$.

Solution: $\displaystyle\lim_{h\to 0} k(-1) = k(-1) = k(1) = 8$ and $\displaystyle\lim_{h\to 0} \frac{k(3+h)-k(3)}{h} = k'(3) = -2$, so

$$\lim_{h\to 0} k(-1) + \lim_{h\to 0}\frac{k(3+h)-k(3)}{h} \;=\; 8+(-2) \;=\; 6.$$

Answer: 6

d. [2 points] Use the table to estimate $k''(4.5)$.

Solution: $\displaystyle k''(4.5) \;\approx\; \frac{k'(5)-k'(4)}{5-4} \;=\; \frac{-5-(-1)}{5-4} \;=\; -4.$

Answer: -4

This problem continues on the next page.

This problem continues from the previous page. The table of some values of the even function $k(x)$ and its derivative $k'(x)$ is displayed again for convenience. Recall that $k(x)$, $k'(x)$, and $k''(x)$ are continuous on $(-\infty, \infty)$, and $k(x)$ is decreasing on $(0, \infty)$. Your final answers in this problem should not include the letter k.

x	0	1	2	3	4	5
$k(x)$	12	8	7	2	0	-3
$k'(x)$	0	-3	-4	-2	-1	-5

e. [2 points] Find the linear approximation $L(x)$ of the function $j(x) = 3k(2x) + 1$ at the point $x = 2$.

Solution: We have $j(2) = 3k(4) + 1 = 3 \cdot 0 + 1 = 1$ and $j'(2) = 6k'(4) = -6$, so

$$L(x) = j(2) + j'(2)(x-2) = 1 - 6(x-2) = -6x + 13.$$

Answer: $L(x) =$ $1 - 6(x-2)$

f. [2 points] Use a right-hand Riemann sum with two equal subdivisions to estimate $\int_1^5 k(x)\,dx$. Write out all the terms in your sum, which you do not need to simplify.

Solution:

$$2 \cdot k(3) + 2 \cdot k(5) = 2 \cdot 2 + 2 \cdot (-3) = -2.$$

g. [2 points] Does the sum described in part **f**. *overestimate*, *underestimate*, or *equal* the value of

$$\int_1^5 k(x)\,dx?$$

Circle your answer and provide a brief explanation. If there is not enough information to decide, circle NEI.

Circle one: OVERESTIMATE   [UNDERESTIMATE]   EQUAL   NEI

Explanation:

Solution: $k(x)$ is decreasing, and a right-hand Riemann sum of a decreasing function always underestimates the corresponding integral.