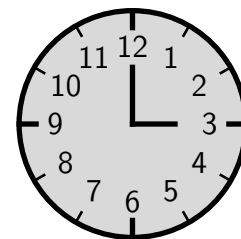


10. [8 points]

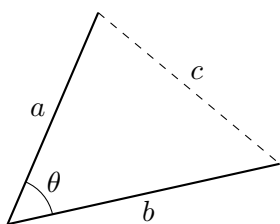
The hour and minute hands of a certain clock are 3 and 4 inches long, respectively. Suppose the hands move continuously in the clockwise direction, with the minute hand making one complete revolution every hour and the hour hand making one complete revolution every 12 hours.



- a. [2 points] If θ is the radian measure of the (smaller) angle between the two hands, what is $\frac{d\theta}{dt}$ at 3pm, in radians per hour?

Solution: Since the minute hand moves at 2π radians per hour and the hour hand moves at $\frac{\pi}{6}$ radians per hour, at 3pm the angle between them is changing at a rate of

$$\frac{d\theta}{dt} = \frac{\pi}{6} - 2\pi = -\frac{11\pi}{6} \text{ radians per hour.}$$



Answer: $\frac{d\theta}{dt} = \underline{\hspace{2cm} -\frac{11\pi}{6} \hspace{2cm}}$

In part b., you may want to use the *Law of Cosines*, which states that in a triangle where θ is the angle between two sides of length a and b , the length c of the third side satisfies the equation $c^2 = a^2 + b^2 - 2ab \cos \theta$. (See the picture to the left.)

- b. [5 points] At what rate is the distance between the tips of the hour and minute hands on the clock changing at 3pm? *Show all of your work, and include units.*

Solution: Let z be the distance between the tips of the hour and minute hands, so

$$z^2 = 3^2 + 4^2 - (2)(3)(4) \cos \theta = 25 - 24 \cos \theta.$$

Differentiating with respect to t , we get

$$2z \frac{dz}{dt} = 24 \sin \theta \frac{d\theta}{dt}.$$

At 3pm we have $\theta = \frac{\pi}{2}$, so $\sin \theta = 1$, and $z^2 = 3^2 + 4^2$, so $z = 5$. Also we know $\frac{d\theta}{dt} = -\frac{11\pi}{6}$ from part a. Substituting these values gives us

$$2 \cdot 5 \cdot \frac{dz}{dt} = 24 \cdot 1 \cdot \frac{11\pi}{-6},$$

so

$$\frac{dz}{dt} = \frac{24 \cdot 11\pi}{-2 \cdot 5 \cdot 6} = -\frac{22\pi}{5}.$$

Thus the distance is *decreasing* at a rate of $\frac{22}{5\pi}$ inches per minute.

Answer: The distance is INCREASING **DECREASING** at a rate of $\underline{\hspace{2cm} \frac{22}{5\pi} \text{ inches per minute} \hspace{2cm}}$.

- c. [1 point] Circle *all* times below at which the rate of change of the distance between the tips of the hour and minute hands is equal to zero, or else circle NONE OF THESE if there are none.

12pm (noon)

6pm

9pm

NONE OF THESE