4. [6 points] The expressions below define five different functions of x, labeled i. – v., which are used for answers in parts **a**. and **b**.

i. $\dfrac{x^2+1}{x-3}$ ii. $\dfrac{x^2-1}{x^3-8}$ iii. $\dfrac{x}{e^x}$ iv. $\dfrac{e^x}{x}$ v. $\dfrac{(x-2)^{1/3}}{x}$

a. [2 points] Which of the above functions have a limit of 0 as $x \to \infty$? *Circle all correct answers.*

i. [ii.] [iii.] iv. [v.] NONE OF THESE

b. [2 points] Which of the above functions satisfy the hypotheses of the Mean Value Theorem on the interval $[1,3]$? *Circle all correct answers.*

i. ii. [iii.] [iv.] v. NONE OF THESE

c. [2 points] Circle the *one correct equality below* to complete the statement of the Mean Value Theorem applied to the function $f(x) = e^x/x$ on the interval $[1,3]$:

"There is a point c in the interval $(1,3)$ such that ..."

$\dfrac{xe^x - e^x}{x^2} = \dfrac{\frac{1}{3}e^c - e}{2}$ $\dfrac{ce^c - e^c}{c^2} = \dfrac{e^3}{9}$ $\dfrac{e^c}{c} = \dfrac{\frac{1}{3}e^3 - e}{3-1}$ $\boxed{\dfrac{e^c(c-1)}{c^2} = \dfrac{e^3}{6} - \dfrac{e}{2}}$

5. [5 points] Find the global maximum and minimum output values of the function

$$y = w(x) = 3 - 2x^2 + \frac{x^4}{2}$$

on the interval $[-1,2]$, and list *all* x-values at which the global maximum and minimum occur. *Show all your work, and use calculus to justify your answers, which should be numerical.*

Solution: First we find the critical points of $w(x)$. Since $w(x)$ is differentiable everywhere, in order to find the critical points we just have to solve $w'(x) = 0$. We have

$$w'(x) = -4x + 2x^3 = 2x(x^2-2) = 2x(x-\sqrt{2})(x+\sqrt{2}).$$

Thus the critical points of $w(x)$ are $x = 0, \pm\sqrt{2}$. Now we evaluate $w(x)$ at the endpoints of $[-1,2]$, and at its critical points that belong to $[-1,2]$. We have

$$w(-1) = \frac{3}{2}, \qquad w(0) = 3, \qquad w(\sqrt{2}) = 1, \qquad w(2) = 3.$$

The greatest of these values is the maximum of $w(x)$ on $[-1,2]$, and the least is the minimum. So, on the interval $[-1,2]$, $w(x)$ has a max of $y = 3$ which occurs at $x = 0$ and $x = 2$, and a min of $y = 1$ which occurs at $x = \sqrt{2}$.

Answer: On the interval $[-1,2]$, the function $w(x)$ has a ...

global maximum value of $y =$ 3, occuring at the point(s) $x =$ 0, 2,

and a global minimum value of $y =$ 1, occuring at the point(s) $x =$ $\sqrt{2}$.