7. [9 points] In an unexpected twist, Carson Soltonni also runs a business selling vacuum cleaners out of his house. The cost in hundreds of dollars for him to produce q hundred vacuum cleaners is

$$C(q) = \frac{q^3}{3} - 5q^2 + 59q + 5.$$

Carson sells his vacuum cleaners for 50 dollars each, and he is trying to determine how many to sell in order to maximize profit. Some values of $C(q)$, rounded to the nearest integer, are given below.

q	1	2	3	4	5	6	7	8	9
$C(q)$	59	106	146	182	217	251	287	328	374

a. [1 point] What is the fixed cost of Carson's business?

Answer: 5 hundred dollars.

b. [3 points] Find the marginal revenue function $MR(q)$ and marginal cost function $MC(q)$ of Carson's business, in hundreds of dollars per hundred vacuum cleaners.

Solution: The marginal revenue and marginal cost functions are the derivatives of the revenue and cost functions, respectively. Since Carson sells vacuum cleaners for 50 dollars each, his marginal revenue is \$50 per vacuum cleaner, or, equivalently, 50 hundred dollars per hundred vacuum cleaners. And the marginal cost function will be $MC(q) = C'(q) = q^2 - 10q + 59$.

Answer: $MR(q) =$ 50 and $MC(q) =$ $q^2 - 10q + 59$

c. [3 points] How many vacuum cleaners should Carson produce and sell to maximize profit? *Show your work and use calculus.* You do not need to fully justify your answer, but partial credit may be awarded for work shown.

Solution: Carson's profit, $\pi(q)$, is equal to revenue minus cost, that is, $\pi(q) = R(q) - C(q)$. We want to maximize $\pi(q)$ over the interval $[0, \infty)$. Since $\pi(q)$ is differentiable everywhere, its critical points will occur when $\pi'(q) = 0$, that is, when $MR(q) = MC(q)$. We have

$$\pi'(q) = MR(q) - MC(q) = 50 - (q^2 - 10q + 59) = -(q^2 - 10q + 9) = -(q-1)(q-9),$$

so the critical points of $\pi(q)$ occur at $q = 1$ and $q = 9$. Checking the endpoints, we have

$$\pi(0) = -5 \qquad \text{and} \qquad \lim_{q\to\infty} \pi(q) = \lim_{q\to\infty} (50q - C(q)) = -\infty.$$

Plugging the critical points into π, we get

$$\pi(1) = 50 - 59 = -9 \qquad \text{and} \qquad \pi(9) = 450 - 374 > 0.$$

This is enough to show that profit is maximized at $q = 9$, that is, when Carson produces and sells 900 vacuum cleaners.

Answer: 9 hundred vacuum cleaners.

d. [2 points] Unsure how to solve the calculus problem in part **c.**, Carson just decides to produce and sell as many vacuum cleaners as he can. Unfortunately, a court order terminates Carson's business immediately after he had produced and sold 600 vacuum cleaners. At this point, had Carson's business *gained* or *lost* money? How much?

Give your answer by circling GAINED *or* LOST *and writing a* positive *number on the blank.*

Solution: Selling 600 vacuum cleaners at \$50 per unit nets Carson \$30,000. On the other hand, the cost of selling 600 vacuum cleaners is $C(6) = 251$ hundred dollars, or \$25,100. Thus Carson has gained $30000 - 25100 = 4900$ dollars after selling 600 vacuum cleaners.

Answer: Carson's business **[GAINED]** LOST 49 hundred dollars.