9. [10 points] A new internet meme has gone viral, and you are attempting to quantify its popularity. Let $V(m)$ model the total number of people who have *seen* the meme, and $L(m)$ the number of people who currently *like* the meme, m days after the meme first appeared. Assume the functions $V(m)$ and $L(m)$ have continuous derivatives, and that $V(m)$ is increasing. Also assume $L(0) = V(0) = 0$ and that everyone who likes the meme has seen it, but that people can change their minds over time about whether they like the meme.

a. [2 points] Circle the **one best** practical interpretation of the equation

$$\int_0^7 V'(m)\,dm \;=\; L(7).$$

i. A week after the meme first appeared, everyone who has seen it likes it.

ii. The number of people who like the meme 7 days after it first appeared is the average rate of change of $V(m)$ over that week.

iii. The average number of new people per day who saw the meme over the first week since it appeared is equal to the number of people who liked it at the end of that week.

iv. The integral of the rate at which new people saw the meme over the first week since it appeared is equal to the net change in the number of new people who saw it that week.

Note: each of the parts **b**.–**d**. *below has at least one correct answer.*

b. [2 points] Circle **all equations** below that support the following statement: "Three weeks after the meme first appeared, about one new person is seeing it every second!"

(i) $V'(3) = 1$ (ii) $V'(21) = 24 \cdot 60^2$

(iii) $(V^{-1})'\big(V(21)\big) = \frac{1}{24 \cdot 60^2}$ (iv) $(V^{-1})'(24 \cdot 60^2) = \frac{1}{V'(21)}$

c. [3 points] Circle **all** statements below that *must be true* if $\int_{20}^{27} L'(m)\,dm = -7$ million.

i. The number of people who liked the meme 20 days after it first appeared was greater than the number of people who liked the meme 27 days after it first appeard.

ii. At least one person who had previously liked the meme later changed their mind and decided they *didn't* like it.

iii. There is some number m for which $L'(m) = -1{,}000{,}000$.

iv. Twenty days after the meme first appeared, at least 7 million people had seen it.

d. [3 points] One noted expert in meme popularity argues that opinions change and memes fade over time, so she measures meme popularity by awarding one "popularity point" for *each day that one person likes a meme*. For example, one person liking a meme for three days would generate 3 popularity points, as would six people liking the meme for half a day each.

Circle **all** expressions below that give the total number of "popularity points" our meme would receive in the first three weeks after it first appeared, according to this expert's model.

(i) $L(21)$ (ii) $21 \cdot L(21)$ (iii) $\frac{L(21)-L(0)}{21-0}$

(iv) $\int_0^{21} L(t)\,dt$ (v) $\int_0^{21} L'(t)\,dt$ (vi) $\frac{1}{21}\int_0^{21} L'(t)\,dt$