8. [9 points] Consider the family of functions given by

$$P(t) = \frac{A}{1 + Be^{-kt}}$$

where A, B, and k are **positive** constants. The first and second derivatives of $P(t)$ are

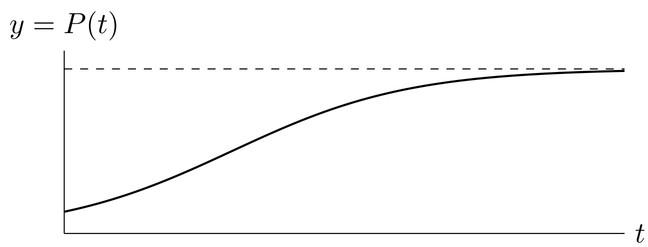
$$P'(t) = \frac{ABke^{kt}}{(B + e^{kt})^2} \qquad \text{and} \qquad P''(t) = \frac{ABk^2e^{kt}(B - e^{kt})}{(B + e^{kt})^3}.$$

a. [3 points] Find all zeros of $P'(t)$ and $P''(t)$. Your answers may involve the constants A, B, and k. If there are none of a particular type, write NONE. *Hint: Remember that A, B, and k are just positive constants.*

Answer: $P'(t)$ has zero(s) at $t =$ ____________________

$P''(t)$ has zero(s) at $t =$ ____________________

Researchers have demonstrated that, for appropriate values of A, B, and k, the function $P(t)$ is a good model for the total amount of oil produced in the US over the t years since 1950, in billions of barrels. For these particular values, a graph of $P(t)$ for $t \geq 0$ is shown below, where $t = 0$ corresponds to the start of 1950.



It is known or estimated that

- $\lim_{t\to\infty} P(t) = 180$, that is, US oil reserves would be depleted after using 180 billion barrels,
- at the start of 1950, a total of 40 billion barrels of oil had been produced in the US, and
- $P(t)$ was increasing the fastest, that is, the rate of oil production was largest, in 1970.

b. [6 points] Find the exact values of A, B, and k for this model. You do not need to simplify.

Answer: $A =$ ____________________ $B =$ ____________________ $k =$ ____________________