1. [13 points] Given below is a table of values for the **decreasing** function $H(w)$ and its derivative, $H'(w)$. Suppose the functions $H(w)$, $H'(w)$, and $H''(w)$ are all defined and continuous on $(-\infty, \infty)$.

w	0	1	2	3	4	5	6	7	8	9	10
$H(w)$	15	13	12	10	9	7	6	5	3	1	0
$H'(w)$	0	-1	-5	-3	-3	-2	-1	0	-1	0	-2

a. [3 points] Use a right–hand Riemann sum with **five** equal subdivisions to estimate $\int_0^{10} H(w)\,dw$. Write out all the terms in your sum. You do not need to simplify, but your answer should not include the letter H.

Solution:

$$\begin{aligned} 12 \times 2 + 9 \times 2 + 6 \times 2 + 3 \times 2 + 0 \times 2 &= \\ 2\Big(12 + 9 + 6 + 3 + 0\Big) &= 60 \end{aligned}$$

b. [1 point] Does the answer to part **a.** overestimate, underestimate, or equal the value of $\int_0^{10} H(w)\,dw$? Circle your answer. If there is not enough information, circle NEI. You do not need to show any work for this part of the problem.

OVERESTIMATE $\boxed{\text{UNDERESTIMATE}}$ EQUAL NEI

c. [2 points] How many equal subdivisions of $[0, 10]$ are needed so that the difference between the left and right Riemann sum approximations of $\int_0^{10} H(w)\,dw$ is exactly 1.5?

Solution:

$$\begin{aligned} \Big(H(10) - H(0)\Big)\frac{(b-a)}{n} &= R - L \\ \frac{(15-0)(10-0)}{n} &= 1.5 \\ n &= 100. \end{aligned}$$

Answer: 100

1. (continued) *The information from the problem is repeated for convenience.*

Given below is a table of values for the **decreasing** function $H(w)$ and its derivative, $H'(w)$. Suppose the functions $H(w)$, $H'(w)$, and $H''(w)$ are all defined and continuous on $(-\infty, \infty)$.

w	0	1	2	3	4	5	6	7	8	9	10
$H(w)$	15	13	12	10	9	7	6	5	3	1	0
$H'(w)$	0	-1	-5	-3	-3	-2	-1	0	-1	0	-2

In **d.–f.**, give numerical answers.

d. [2 points] Find the average rate of change of $H'(w)$ on the interval $[3, 7]$.

Solution:

$$\frac{H'(7) - H'(3)}{7-3} = \frac{0-(-3)}{7-3} = \frac{3}{4}$$

Answer: 3/4

e. [2 points] Use the table to estimate $H''(1.5)$.

Solution:

$$\frac{H'(2) - H'(1)}{2-1} = \frac{-5-(-1)}{2-1} = -4$$

Answer: -4

f. [3 points] Find $\displaystyle\int_2^5 \big(3H'(w) - 2w\big)\, dw$.

Solution:

$$\begin{aligned}\int_2^5 \big(3H'(w) - 2w\big)\, dw &= 3\int_2^5 H'(w)\, dw - 2\int_2^5 w\, dw \\ &= 3\big(H(5) - H(2)\big) - w^2\Big|_2^5 \\ &= 3(7-12) - (5^2 - 2^2) = -36\end{aligned}$$

Answer: -36