**11.** [7 points] Two birds, Tarik and Clara, are collecting materials for their nest. Clara is flying straight from the nest at point A to the pond at point B, and Tarik is flying straight from the forest at point C toward the nest at point A. At the moment shown in the diagram below, Tarik spots Clara flying toward the pond, and Clara is 40 meters from the nest, while Tarik is 30 meters from the nest. Note that each bird is represented in the figure by a triangle, pointing in the direction they are flying.
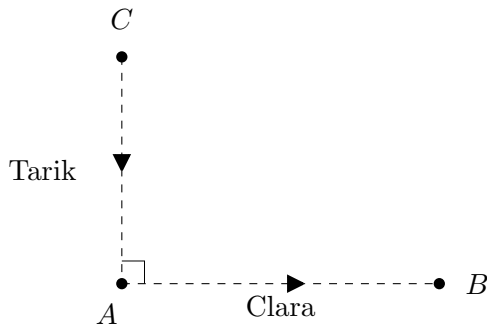
a. [2 points] How far apart are Tarik and Clara at the moment he spots her? **Include units.**

Solution: By the Pythagorean Theorem, the distance between Tarik and Clara at the moment he spots her is
$$\sqrt{30^2 + 40^2} \;=\; \sqrt{900 + 1600} \;=\; \sqrt{2500} \;=\; 50 \text{ meters.}$$

Answer: _____ 50 **meters** _____

b. [5 points] If Tarik's speed is 6 meters per second and Clara's is 4 meters per second at the moment he spots her, how fast is the distance between them changing at this moment? Give your answer exactly, and **include units.** Are they getting closer to each other or farther away from each other at this moment? Circle your answer below. You must justify your answer through your work to receive credit for the answer you circle.

Solution: We view the dashed lines in the figure above as coordinate axes with point A as the origin, and let x represent Clara's position between A and B, and y Tarik's position between A and C. So at the moment Tarik spots Clara, we have $x = 40$, $y = 30$, $\frac{dx}{dt} = 4$, and $\frac{dy}{dt} = -6$. Letting z be the distance between Tarik and Clara, we have that x, y, and z all depend on t and satisfy $x^2 + y^2 = z^2$ by the Pythagorean Theorem. differentiating this equation with respect to t gives us

$$2z\frac{dz}{dt} \;=\; 2x\frac{dx}{dt} + 2y\frac{dy}{dt}.$$

Plugging in $x = 40$, $y = 30$, $\frac{dx}{dt} = 4$, and $\frac{dy}{dt} = -6$, along with $z = 50$ from part a., we get

$$100\frac{dz}{dt} \;=\; 2(40)(4) + 2(30)(-6),$$

and solving this for $\frac{dz}{dt}$ gives

$$\frac{dz}{dt} \;=\; \frac{320 - 360}{100} \;=\; -\frac{2}{5}.$$

This means that, at the given moment, the distance between the two birds is *decreasing* at a rate of $\frac{2}{5}$ meters per second.

Answer: _____ $-2/5$ **meters per second** _____

The birds are getting: **closer to each other**   **farther away from each other**