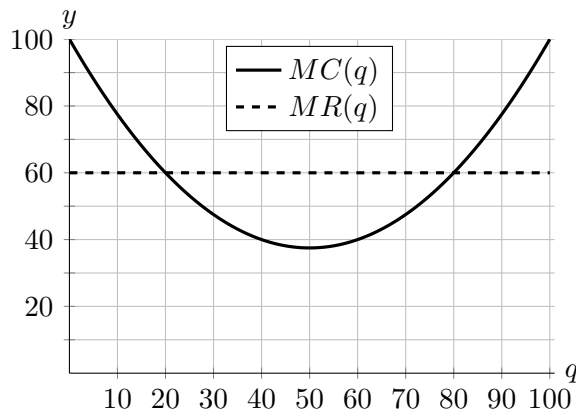


9. [10 points] Olivia knits and sells custom-made clothing online, and she is considering adding hats, scarves, and mittens to her inventory this winter. She has modeled the expected cost and revenue for each of these items, and wants to know how many of each she should produce in order to maximize profit. In each part below, help her decide based on the given information.

a. [2 points]

Olivia can make at most 100 hats, and she has calculated the marginal cost $MC(q)$ and marginal revenue $MR(q)$ of producing and selling q hats, in dollars per hat, as shown in the graphs on the right.

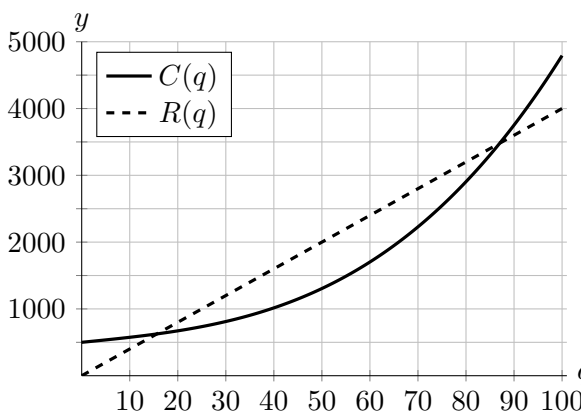


How many hats should she make in order to maximize profit? *Circle the one correct answer below.*

0 20 50 80 100 NONE OF THESE

b. [2 points]

Olivia can make at most 100 scarves, and she has calculated the cost $C(q)$ and revenue $R(q)$ of producing and selling q scarves, in dollars, as shown in the graphs on the right.



Approximately how many scarves should she make in order to maximize profit?

Circle the one correct answer below.

0 17 55 87 100 NONE OF THESE

- c. [6 points] Olivia can make at most 300 pairs of mittens, and she has calculated the cost $C(q)$ and revenue $R(q)$ of producing and selling q hundred pairs of mittens, in hundreds of dollars, to be

$$R(q) = 30q \quad \text{and} \quad C(q) = 2q^3 - 9q^2 + 42q.$$

How many hundreds of pairs of mittens should Olivia make in order to maximize profit? *Show all your work and use calculus to find your answer.*

Solution: Profit, $\pi(q)$, is equal to revenue minus cost, so

$$\pi(q) = R(q) - C(q) = 30q - (2q^3 - 9q^2 + 42q) = -2q^3 + 9q^2 - 12q.$$

In order to maximize $\pi(q)$ on the interval $[0, 300]$, we differentiate $\pi(q)$ to find its critical points:

$$\pi'(q) = -6q^2 + 18q - 12 = -6(q^2 - 3q + 2) = -6(q - 1)(q - 2).$$

So the critical points of $\pi(q)$ are $q = 1$ and $q = 2$, and now we just calculate $\pi(q)$ at these critical points and at the endpoints of our interval:

$$\pi(0) = 0, \quad \pi(1) = -5, \quad \pi(2) = -4, \quad \pi(3) = -9.$$

This means Olivia should not make any mittens!

Answer: _____ 0 _____ hundreds of pairs of mittens