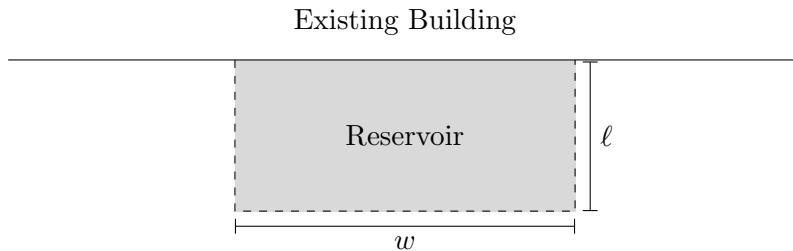


## 4. [14 points]

A city is building a new water treatment facility. A water reservoir will be built next to an existing building as shown here (viewed from above):



Three walls of the reservoir (dashed) and the floor of the reservoir (shaded) will be made of concrete. The cost for the concrete on the three walls is \$500 per meter of perimeter and the cost for the concrete for the floor is \$1000 per square meter.

a. [3 points] Find a formula for the cost  $C$  for the concrete to build the reservoir if it has dimensions  $\ell$  and  $w$  (both in meters) as shown above.

**Answer:**  $C = \underline{\hspace{2cm}}$

b. [2 points] If the budget for the total cost of the concrete is \$5,000,000, write a formula for the length  $\ell$  of the reservoir in terms of its width  $w$ . This relationship can be used to find the largest reservoir possible, given the concrete budget, but you do not need to find this.

**Answer:**  $\ell = \underline{\hspace{2cm}}$

4. (continued) The city is also looking for the most cost-effective way to build a large tank in the shape of a cylinder for the water treatment facility. The cost, in dollars, of a tank with radius  $r$  and height  $h$  (both given in meters) is given by

$$50\pi r^2 + 80\pi r h.$$

c. [2 points] The tank must have volume 10,000 cubic meters, so that  $\pi r^2 h = 10,000$ . Given this constraint, find a formula for the cost  $T(r)$  of the tank, in dollars, that is a function of the variable  $r$  only. *Your answer should not include the height  $h$  of the tank.*

**Answer:**  $T(r) =$  \_\_\_\_\_

d. [2 points] Note that, in context,  $T(r)$  has a domain of  $(0, \infty)$ . Determine  $\lim_{r \rightarrow 0^+} T(r)$  and  $\lim_{r \rightarrow \infty} T(r)$ . Each answer should either be a number, or  $\infty$  or  $-\infty$ .

$$\lim_{r \rightarrow 0^+} T(r) =$$
 \_\_\_\_\_

$$\lim_{r \rightarrow \infty} T(r) =$$
 \_\_\_\_\_

e. [5 points] Find the radius that will minimize the cost of the tank. Use calculus, and show your work, but you need not simplify your numerical answers. *Make sure to justify why your answer is the global minimum; you may use work from previous parts of this problem.*

**Answer:** Cost is minimized when  $r =$  \_\_\_\_\_