1. [12 points] You are moving straight upward in a hot air balloon.

- Your altitude $a = h(t)$, in miles, is a function of your time t, in hours, since takeoff.
- The air temperature $T = m(a)$ outside your balloon, in degrees Fahrenheit (°F), is a function of your altitude a, in miles.

Both h and m are differentiable and invertible. The following values are known.

- $h(0.5) = 0.33$
- $h'(0.5) = 2$
- $m(0.33) = 55$
- $m'(0.33) = -16$
- $h(1) = 0.5$
- $h'(1) = 0.33$
- $m(0.5) = 52.8$
- $m'(0.5) = -15$

a. [5 points] For each of the following two equations: fill in the missing value, then use a complete sentence to interpret the equation practically.

$m(h(0.5)) =$ 55

Interpretation:

Solution: Half an hour after takeoff, the air temperature outside your balloon is 55°F.

$\int_{0.5}^{1} h'(t)\,dt =$ 0.17

Interpretation:

Solution: In the second half hour of your flight, you ascend an additional 0.17 miles.

b. [3 points] Recall that $m(0.33) = 55$ and $m'(0.33) = -16$. Use these two values to estimate the temperature at an altitude of 0.43 miles. Include units and show your work.

Solution: $m'(0.33) = -16$ tells us that the temperature at an altitude of 0.43 miles is about 1.6°F lower than the temperature at an altitude of 0.33 miles. Then, since we were told that $m(0.33) = 55$, we know that $m(0.43)$ should be about $55 - 1.6 \approx 53.6$°F. Note that one can also set up and use a formal linear approximation here.

Answer (include units): $\approx$ 53.6 °F

c. [4 points] How fast is the air temperature outside your balloon decreasing 30 minutes after takeoff? That is, at what rate is the air temperature decreasing as a function of **time** at this instant? Include units, and show your work to justify your answer.

Answer (include units): 32°F per hour

Solution: We are looking for the derivative of $m(h(0.5))$, which, by the chain rule, is $m'(h(0.5)) * h'(0.5) = m'(0.33) * 2 = -32$. So the temperature is decreasing at 32°F per hour. Note that the first given value tells us that, half an hour after takeoff, your altitude is 0.33 of a mile, so this computation is equivalent to multiplying the rate at which the temperature is changing with altitude, $m'(0.33) = -16$ °F per mile, by the rate at which altitude is changing with time, $h'(0.5) = 2$ miles per hour.