2. [8 points] Consider the family of functions given by $h(x) = ax^{2/3} + bx$ with parameters a and b. It may be helpful to recall that $8^{1/3} = 2$ and $64^{1/3} = 4$.

a. [4 points] Find values of a and b so that $h(x)$ has a critical point at $(8, 4)$.

Solution: $h(x)$ will have a critical point when $h'(x) = 0$ or $h'(x)$ is undefined. We have $h'(x) = \frac{2a}{3x^{1/3}} + b$, which gives us $h'(8) = \frac{2a}{3 \cdot 8^{1/3}} + b = 0$, so $a = -3b$. We also know that $h(8) = 4$, so substituting $-3b$ for a, we get the equation

$$-3b(8)^{2/3} + 8b = 4$$

$$-4b = 4$$

$$b = -1.$$

This tells us that $a = 3$.

Answer: $a =$ ___3___ $b =$ ___-1___

b. [4 points] Given the values you of a and b that you found in the previous part, classify all local extrema of $h(x)$. If there aren't any local extrema of a particular type, write NONE. **Be sure you show enough evidence** to support your conclusions.

Solution: Using part (a), we have that $h(x) = 3x^{2/3} - x$ and $h'(x) = \frac{2}{x^{2/3}} - 1 = \frac{2 - x^{1/3}}{x^{1/3}}$. We then have two critical points: $x = 8$ nad $x = 0$. We get the following sign chart:

$h'(x)$: $\frac{+}{-} = -$ | 0 | $\frac{+}{+} = +$ | 8 | $\frac{-}{+} = -$

We then get that $x = 0$ is a local minimum and $x = 8$ is a local maximum.

Answer: Local Min(s): $x =$ ___0___ Local Max(es): $x =$ ___8___