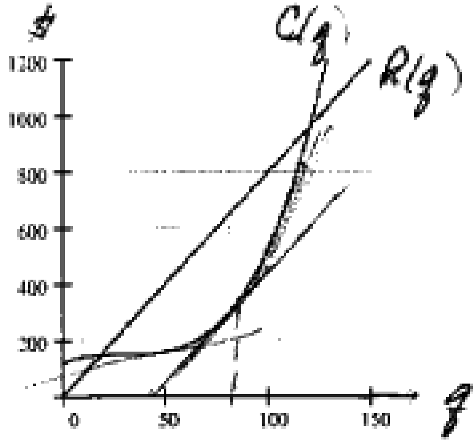
(10.) Cost and revenue functions for a charter bus company are shown in the figure below, where q is the number of buses that the company owns.



(a) **(4 pts)** Should the company add a 50^{th} bus? How about a 100^{th}? Explain your answers using marginal revenue and marginal cost. (You may illustrate your reasons graphically as well, if you like.)

The company should add the 50^{th} bus, because at $q = 50$ MC (or the slope of $C(q)$) is less than MR -- ie, revenue is increasing faster than cost. The company should not add the 100^{th} bus because @ $q = 100$, costs are increasing at a faster rate than revenue -- ie $MR < MC$.

(b) **(3 pts)** What does $C'(50) = A$ (A, a constant) mean in the context of this problem? What are the units of the 50 and the units of A?

In the expression $C'(50) = A$, the units of 50 are buses & the units of A are dollars per bus. The expression represents the rate at which costs are increasing once the company has 50 buses. In economic terms, approximately the cost of going from 50 - 51 buses is $\$A$.

(c) **(4 pts)** Estimate the number of buses the company should have in order to maximize profit. Explain how you determined your estimate.

Profit will be maximized when $R(q) - C(q)$ is greatest -- which also (by calculus) occurs when $R'(q) = C'(q)$. This means the slope of the tangent to $C(q)$ is equal to the slope of $R(q)$. This appears to be around $q = 80$. [See graph.]

(Problem 10 continued)

(d) (6 pts)

(i) If the average cost, $a(q)$, is given by $a(q) = \frac{C(q)}{q}$, approximate q_0 so that $a(q_0)$ is the minimal average cost.

From the graph, minimum average cost appears to be when $q \approx 60$.

(ii) Show ***analytically*** that average cost will be minimized when $C'(q) = a(q)$.

Given $a(q) = \frac{C(q)}{q}$, then

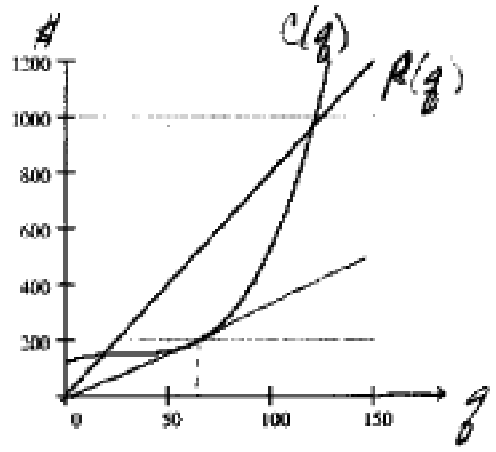
$$a'(q) = \frac{qC'(q) - C(q)(1)}{q^2} \quad \& \quad a'(q) = 0 \text{ if}$$

$$qC'(q) = C(q)$$

$$\rightarrow \quad C'(q) = \frac{C(q)}{q} = a(q)$$

Note: $a(q) = \frac{C(q)}{q}$ can be visualized as slope from $(0,0)$ to $(q, C(q))$. These slopes decrease for $0 < q < q_0$ & then increase for $q > q_0$. Thus, there is a min @ $q = q_0$.

(iii) Demonstrate on the graph below how this result can be shown graphically.



(11.) And, for good measure, one last derivative.... No need to simplify, but **show all your work.**

(3 pts) Find the derivative of $k(t) = \frac{(3t-4)}{\cos(2t)}$.

ok to here

$$k'(t) = \frac{\cos(2t)(3) - (3t-4)(-\sin 2t)(2)}{\cos^2(2t)}$$

$$\underline{\text{or}} \quad = \frac{3\cos(2t) + (6t-8)\sin(2t)}{\cos^2(2t)}$$

If use Quot Rule get

$k'(t) = (3t-4)\left[-(\cos(2t))^{-2}(-2\sin(2t))\right] + 3(\cos(2t))^{-1}$

[One more page...]