

(Problem 10 continued)

(d) (6 pts)

- (i) If the average cost, $a(q)$, is given by $a(q) = \frac{C(q)}{q}$, approximate q_0 so that $a(q_0)$ is the minimal average cost.

From the graph, minimum average cost appears to be when $q \approx 60$.

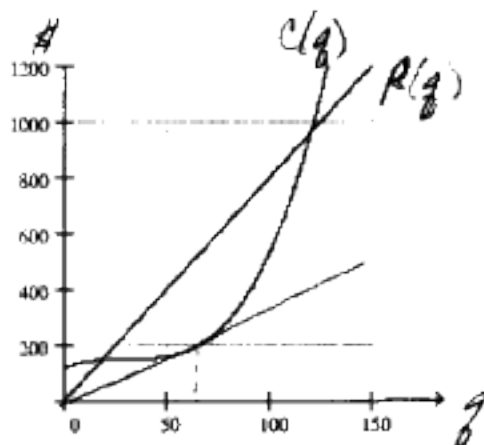
- (ii) Show *analytically* that average cost will be minimized when $C'(q) = a(q)$.

Given $a(q) = \frac{C(q)}{q}$, then

$$a'(q) = \frac{qC'(q) - C(q)}{q^2} = 0 \text{ if } qC'(q) = C(q) \rightarrow C'(q) = \frac{C(q)}{q} = a(q)$$

Note: $a(q) = \frac{C(q)}{q}$ can be visualized as slope from $(0,0)$ to $(q, C(q))$. These slopes decrease for $0 < q < q_0$ & then increase for $q > q_0$. Thus, there is a min @ $q = q_0$.

- (iii) Demonstrate on the graph below how this result can be shown graphically.



- (11.) And, for good measure, one last derivative.... No need to simplify, but **show all your work**.

- (3 pts) Find the derivative of $k(t) = \frac{(3t-4)}{\cos(2t)}$.

Q: use Quot Rule get
 $k'(t) = \frac{(3t-4)' \cos(2t) - (3t-4) \cos'(2t)}{\cos^2(2t)}$
 $= \frac{(3) \cos(2t) - (3t-4)(-2 \sin(2t))}{\cos^2(2t)}$
 $= \frac{3 \cos(2t) + (6t-8) \sin(2t)}{\cos^2(2t)}$

$$k'(t) = \frac{\cos(2t)(3) - (3t-4)(-\sin(2t)(2))}{\cos^2(2t)}$$

$$= \frac{3 \cos(2t) + (6t-8) \sin(2t)}{\cos^2(2t)}$$

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