(7.) The function $f(x) = x\ln(x)$ has one critical point on the interval (0, 5).

(a) **(4 pts)** Determine the **exact** x value (i.e., not a decimal approximation) for the location of this critical point.

$$f'(x) = \ln(x) + x\left(\frac{1}{x}\right)$$
$$= \ln(x) + 1$$
$$f'(x) = 0 \text{ if } \ln x = -1$$
$$\text{so } x = e^{-1} = \frac{1}{e}$$

$x = e^{-1}$ or $x = \frac{1}{e}$

(b) **(3pts)** Is this point a maximum or a minimum or neither of these? Explain and show your work. [Use either 1st or 2nd deriv test -- or a good graphical argument.]
By 2nd deriv. test: $f''(x) = \frac{1}{x}$; $f''\left(\frac{1}{e}\right) > 0$
So $x = \frac{1}{e}$ is a local minimum

(c) **(2 pts)** Determine the instantaneous rate of change of f at $x = 1$ and at $x = 2$.

$$f'(1) = \ln(1) + 1 = 1$$
$$f'(2) = \ln(2) + 1 \approx 1.693$$

@ $x = 1$, $f'(1) = 1$

@ $x = 2$, $f'(2) \approx 1.693$

(d) **(2 pts)** What do the values in part (c) suggest about the concavity of the function between $x = 1$ and $x = 2$? Explain.

Since f' increases between $x = 1$ & $x = 2$, this suggests the function is concave up between $x = 1$ & $x = 2$.

(e) **(3 pts)** Determine an equation of the tangent to the graph of f at $x = 1$.

@ $x = 1$, $f(1) = 1 \cdot \ln(1) = 0$; $m = f'(1) = 1$,

so $y - 0 = 1(x - 1)$

$y = x - 1$

(f) **(2 pts)** Use your equation from part (e) to approximate $f(2)$.

Using part (e),
$f(2) \approx 2 - 1 = 1$.

[Note: $f(2) = 2\ln(2) \approx 1.386$]

(g) **(2 pts)** Should your estimate be an underestimate or an overestimate?
Underestimate Why? — Because if the function is concave up, the tangent lies below the function.