4. (9 points) (See problem 3, Chapter 5, Section 1.) Last week David ran the Naked Mile, starting out at a fast pace with the idea of winning the race. His friend, John rode along on his bike to clock David's times. However, at the end of the race the police were chasing David so that he kept on running to avoid being arrested. John followed and recorded David's speeds at 5 minute intervals. David was slowing down all the time, but fortunately for him, the policemen were unable to catch him. They finally gave up chasing him after 25 minutes. David continued for an additional five minutes before stopping.

The speeds John clocked are recorded in the following table. In recounting his experience, David wondered how far he actually ran in the half hour. Help him out by answering the questions in parts (a) and (b). (Be sure to show your work when answering the questions).

time (in minutes):	0	5	10	15	20	25	30
speed (in miles per minute):	.2	.16	.14	.12	.12	.1	.05

(a) Assuming that David's speed never increases throughout the run, use the data in the table to determine the best estimate for the total distance that David ran during the 30 minutes.

Solution: Since David's speed is never increasing, the left hand sum gives an upper estimate for the distance he travelled,

$$\text{Upper est } = (.2 + .16 + .14 + .12 + .12 + .1)\,(5) = 4.2 \quad \text{miles}$$

and the right hand sum gives a lower bound,

$$\text{Lower est } = (.16 + .14 + .12 + .12 + .1 + .05)\,(5) = 3.45 \quad \text{miles}$$

The best estimate, given the above information, is the average of the upper and lower bounds,

$$\frac{4.2 + 3.45}{2} = 3.825 \quad \text{miles}$$

(b) To be sure of estimating the distance David traveled to within .15 miles, how frequently would John have needed to record the measurements of David's speed?

Solution: We want the error to be at most .15 miles. Since the difference between the estimates given by the upper and lower sums with measurements taken Δt minutes apart is $(2-.05)\Delta t$, we want $.15\Delta t \le .15$ or $\Delta t \le 1$ minute. Therefore, if John had taken the measurements every one minute, then we could have been sure of this estimate, no matter which of the estimates obtained in part (a) was used.

If one uses the average of the left and right hand sums for the estimate, however, you can do a little better. In this case, the error taken with measurements Δt minutes apart is $\frac{(2-.05)\Delta t}{2}$, so to make the error at most .15 miles, it suffices to take $.15\Delta t/2 \le .15$ or $\Delta t \le 2$. Therefore, John would only have to measure David's speed every two minutes to achieve this accuracy.