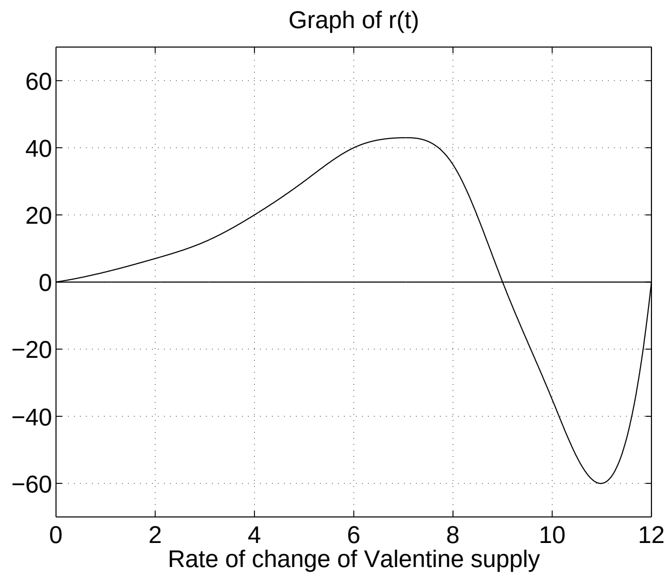
6. (10 points) (See problem 17 and Team problem 26, Chapter 5, Section 3.) Recall that Hankytown, the community famed for making valentines, has a fluctuating population based on the influx of migrant valentine makers. The number of valentines in the city coffers varies also according to the season of the year. The graph below shows the rate, $r(t)$ (in 1000's of valentines per month), at which the supply of valentines changes over a 12 month period, where $t = 0$ corresponds to the beginning of January.



(a) Over what period of time did the valentine supply grow? Solution: The supply grows while $r(t) > 0$ so for about $0 \le t \le 9$. This corresponds to the time from January 1 to the beginning of October.

(b) When was the supply of valentines growing most rapidly?

Solution: The supply was growing most rapidly where $r(t)$ takes its maximum value, or at about $t = 7$, the beginning of August.

(c) Write a mathematical expression giving the average rate at which the valentine supply is changing over the first four months shown in the graph.

Solution: The average value of $r(t)$ over the first four months of the year, $0 \le t \le 4$, is

$$\frac{1}{4}\int_0^4 r(t)\,dt$$

(d) Given that there were 25,000 valentines in the warehouse at the beginning of the period shown, write a mathematical expression for the total number of valentines in the warehouse at the end of the 12 month period. Were there more or fewer than 25,000 valentines at this time. How do you know?

Solution: The number of valentines in the warehouse at the end of the 12 month period (in thousands of valentines) $= 25 + \int_0^{12} r(t)\,dt$. ¿From the graph, it appears that the area under the curve $r(t)$ and above the $t-$axis is greater than the area above the curve $r(t)$ but below the $t-$axis. Therefore, $\int_0^{12} r(t)\,dt > 0$, so there are a greater number of valentines in the warehouse at the end of the period than at the beginning.