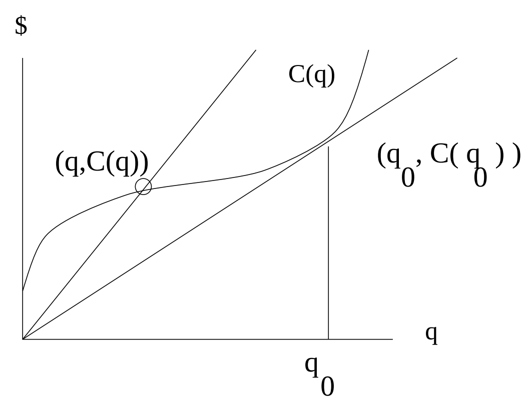
8. (8 points) (See Team problem 16, Chapter 4, Section 4, as well as problems 29 and 31 from Chapter 4, Section 3.) (a) It is a fact from economics that when the average cost $C(q)/q$ of producing $q > 0$ units of a quantity is a minimum, then this average cost is equal to the marginal cost. Show analytically why this is so.

Solution: At a point where the average cost $a(q) = C(q)/q$ is a minimum the derivative of $a(q)$ must be equal to 0. By the quotient rule,

$$a'(q) = \frac{qC'(q) - C(q)}{q^2}, \quad q > 0$$

so $a'(q) = 0$ if and only if $qC'(q) - C(q) = 0$, or $C'(q) = \frac{C(q)}{q}$. Thus, at the critical points of $a(q)$, the marginal cost, $C'(q)$, is equal to the average cost $\frac{C(q)}{q}$.

(b) Using the graph of $C(q)$ shown below, a typical cost function as drawn in the text, indicate on the q-axis the value of q_0 which minimizes the average cost. Explain graphically why the average cost is equal to the marginal cost at this point.



Line through (q,C(q)) has slope equal to average cost.

Line with minimum slope is tangent to graph of C(q).

The average cost $a(q) = \frac{C(q)}{q}$ is the slope of the line from the origin to a point $(q, C(q))$ on the graph of C. The line with minimal slope is shown above. At this point, q_0, the line from the origin is tangent to the curve so its slope is also equal to $C'(q_0)$, the marginal cost at that point.