

11. (9 points) Let $s(t)$ give the position of an object along a straight line at time t and let $v(t)$ denote its instantaneous velocity at time t .

(a) Give the definition of the average velocity of the object over the time interval from $t = a$ to $t = b$.

Solution: The average velocity is

$$\frac{s(b) - s(a)}{b - a}.$$

(b) Give the definition of the average of the velocity function over the interval from $t = a$ to $t = b$.

Solution: The definition of the average of the velocity function is:

$$\frac{1}{b - a} \int_a^b v(t) dt.$$

(c) Is the average velocity of the object over the time interval from $t = a$ to $t = b$ equal to the average of the velocity function over this time interval? If so, explain why. If not, explain why not.

Solution: The two quantities are equal because of the fundamental theorem of calculus. That is, from part (b), the average of the velocity function is equal to

$$\frac{1}{b - a} \int_a^b v(t) dt = \frac{1}{b - a} \int_a^b \frac{ds(t)}{dt} dt.$$

By the fundamental theorem of calculus, the last expression is equal to

$$\frac{1}{b - a} (s(b) - s(a))$$

which, by part a, is the average velocity.

Please rewrite your name and section number.

NAME: _____

SECTION NO: _____