4. (12 points) (a) Give the limit definition of the derivative of a function $f$ at a point $a$.

Solution:

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.$$ 

(b) Use the limit definition of the derivative to find $g'(x)$ for the function $g(x) = 2x^2 - 3x$. [Be sure to show all of your work!]

Solution:

$$g'(x) = \lim_{h \to 0} \frac{(2(x + h)^2 - 3(x + h)) - (2x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2}{h}$$

$$= \lim_{h \to 0} 4x + 2h = 4x - 3$$

(c) Use the Fundamental Theorem of Calculus to find $\int_{2}^{4} (4x - 3) \, dx$. [Note: You must show your work to receive credit.]

Solution: If $g(x) = 2x^2 - 3x$ then $g'(x) = 4x - 3$ so that

$$\int_{2}^{4} (4x - 3) \, dx = \int_{2}^{4} g'(x) \, dx = g(4) - g(2) = 20 - 2 = 18$$

where second equality holds because of the Fundamental Theorem.