**4.** (12 points) **(a)** Give the limit definition of the derivative of a function $f$ at a point $a$.

**Solution:**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

**(b)** Use the limit definition of the derivative to find $g'(x)$ for the function $g(x) = 2x^2 - 3x$. [Be sure to show all of your work!]

**Solution:**

$$\begin{aligned} g'(x) &= \lim_{h \to 0} \frac{\big(2(x+h)^2 - 3(x+h)\big) - \big(2x^2 - 3x\big)}{h} \\ &= \lim_{h \to 0} \frac{\big(2x^2 + 4xh + 2h^2 - 3x - 3h\big) - \big(2x^2 - 3x\big)}{h} \\ &= \lim_{h \to 0} \frac{4xh - 3h + 2h^2}{h} \\ &= \lim_{h \to 0} 4x - 3 + 2h = 4x - 3 \end{aligned}$$

**(c)** Use the Fundamental Theorem of Calculus to find $\displaystyle\int_2^4 (4x - 3)\, dx$. [Note: You must show your work to receive credit.]

**Solution:** *If* $g(x) = 2x^2 - 3x$ *then* $g'(x) = 4x - 3$ *so that*

$$\int_2^4 (4x - 3)\, dx = \int_2^4 g'(x)\, dx = g(4) - g(2) = 20 - 2 = 18$$

*where second equality holds because of the Fundamental Theorem.*