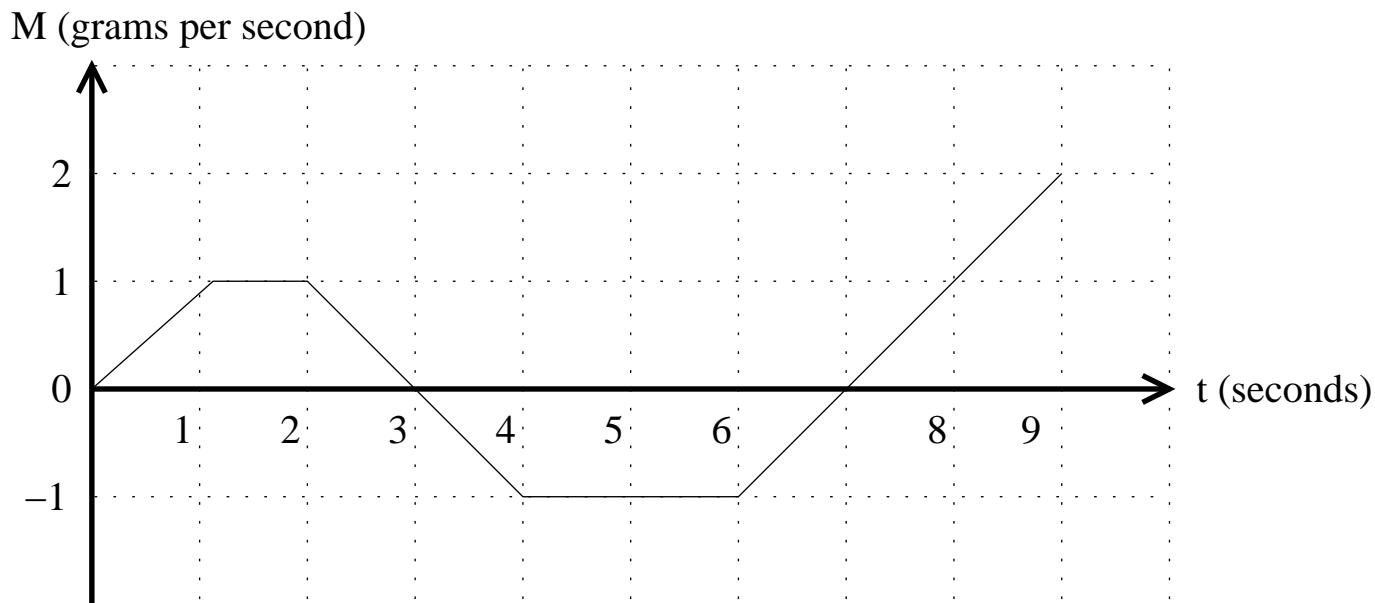


5. (9 points) A substance,  $B$ , is one of several substances involved in a complex chemical reaction. At certain times during this reaction, substance  $B$  is produced by the reaction while at other times it plays the role of a reactant and is consumed. Given that enough reactants are present, the rate  $M$ , of production of substance  $B$  is approximated by the function whose graph is given below.



(a) Over what interval(s) is the amount of substance  $B$  increasing?

**Solution:**  $M = dB/dt$  is positive for  $0 < t < 3$  and for  $7 < t < 9$  so the amount of substance  $B$  present is increasing on those two intervals.

(b) At what time during the reaction is the least amount of substance  $B$  present? Explain.

**Solution:** The least amount of substance  $B$  is present when  $t = 7$ . Because, if  $B(t)$  denotes the amount of  $B$  present  $t$  seconds after the beginning of the reaction, then the change in  $B$ ,  $\Delta B(t) = B(t) - B(0)$  is equal to the integral of  $M$ , the rate of change of  $B$  over the interval from 0 to  $t$ . This shows the amount of  $B$  present increases for  $0 \leq t \leq 3$  by 2 grams, the area under the graph of  $M$  over this interval, so  $B(3) = B(0) + 2$  gms. For  $3 \leq t \leq 7$ , the amount of  $B$  present decreases by 3 grams, the area between the graph of  $M$  and the  $x$ -axis over this interval, so  $B(7) = B(0) - 1$ . And, the amount of  $B$  present then increases for  $7 \leq t \leq 9$  (up to  $B(9) = B(0) + 1$ ). So, the smallest amount occurs when  $t = 7$ .

(c) The reaction takes 9 seconds to complete and will not proceed if there is no substance  $B$  present. There is a value,  $V$ , such that if the reaction begins with  $V$  or fewer grams of substance  $B$ , then the reaction will not proceed to completion. Find the value of  $V$ , and explain your answer.

**Solution:** The value is  $V = 1$ . As explained in in part (b), the least amount of  $B$  is present at  $t = 7$  and is  $B(0) - 1$  gm, one gram less than at the beginning of the reaction. If there had been less than one gram of  $B$  at the beginning, the amount of  $B$  would have been exhausted before  $t = 7$  so the reaction would not have completed.