8. (9 points) The table gives the values of a function obtained from an experiment.

x	0	1	2	3	4	5	6	7	8
$f(x)$	9.3	9.1	8.3	6.9	3.3	-.6	-1.7	-3.5	-6.7

(a) Using these values, estimate $\int_0^8 f(x)\,dx$ using 4 subintervals and right hand endpoints.

Solution: *The interval has length* 8 *and there are to be* 4 *subintervals so* $\Delta x = 8/4 = 2$. *The estimate using right hand sums is then given by:*

$$\sum_{j=1}^{4} f(x_j)\Delta x = f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2 = (8.3 + 3.3 + (-1.7) + (-6.7))\cdot 2 = (3.2)\cdot 2 = 6.4.$$

(b) If f is known to be a decreasing function, can you determine if your answer in part (a) is an over or underestimate? If so, which is it? If not, why not. Be sure to explain your answer.

Solution: *The answer in part* **(a)** *is an underestimate when f is decreasing (see the figure). This is because on each of the subintervals, the value of $f(x)$ is greater than or equal to the value of f at the right hand endpoint, x_j of that subinterval. Therefore, the integral of f over the subinterval is greater than or equal $f(x_j)\Delta x$, the signed area of the rectangle with width Δx and height $f(x_j)$. Note that if $f(x_j) < 0$, then this area is negative, and it is a smaller negative number than the integral of f over this subinterval. Adding up these inequalities over all four subintervals shows that the integral is greater than or equal to the sum of the signed areas of the rectangles, i.e. the right hand sum.*