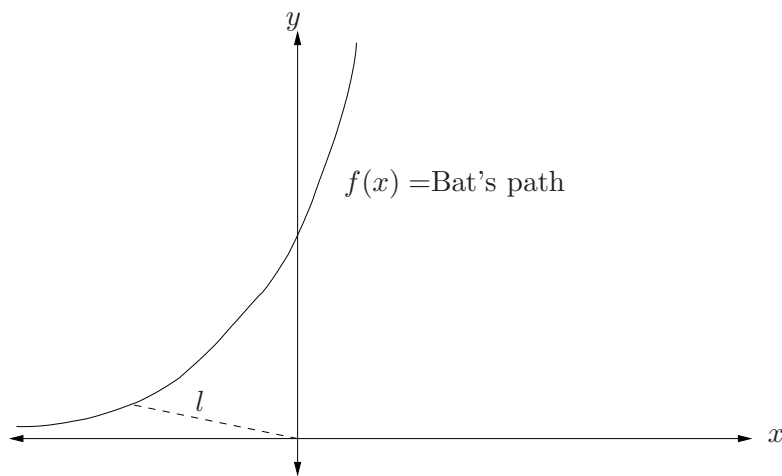


10. (11 points) Hiking through the forest you come upon a cave. As you stand outside the cave and peer in, a bat flies out towards you before veering off into the forest. The bat's path is given in the figure below where the origin represents where you are standing. The distance l represents the distance between you and the bat. Everything is measured in feet.



(a) Find a formula for l^2 in terms of x and $f(x)$.

Using the Pythagorean Theorem one gets

$$l^2 = x^2 + f(x)^2.$$

(b) Let $D = l^2$ and find $\frac{dD}{dx}$.

$$\frac{dD}{dx} = 2x + 2f(x)f'(x)$$

(c) The minimum distance between you and the bat occurs when D is minimized. Find the value of x at this point in terms of $f(x)$ and $f'(x)$.

Setting our answer in part (b) equal to 0, we get that

$$x = -f(x)f'(x)$$

is a critical point. To show this is a minimum, we take the second derivative

$$\frac{d^2D}{dx^2} = 2 + 2[f'(x)]^2 + 2f(x)f''(x).$$

We see from the graph that everything here is positive, so the second derivative of $D(x)$ is positive, so we have indeed found a minimum.

(d) Suppose $f(x) = e^{x+3}$. If a bat comes within 5 feet of you, a panic attack will occur. (Remember that the distance between you and the bat is l , not D !) Did the bat induce a panic attack? [Hint: You are *encouraged* to use your calculator here!]

Using what we found in part (c) we have

$$\begin{aligned}x &= -e^{x+3}e^{x+3} \\ &= -e^{2x+6}.\end{aligned}$$

So we need to solve the equation $x + e^{2x+6} = 0$ for x . We use the calculator to do this, getting the answer $x \approx -2.53$. So then $f(-2.53) = e^{-2.53+3} \approx 1.60$. Looking back at our equation for l^2 , we have $l^2 \approx 8.96$, so $l \approx 3$ ft. Therefore a panic attack does incur!