4. (10 points) Let f be a continuous differentiable function of x. Suppose f is always increasing. The following is a table of values of $f(x)$.

x	.8	.9	1	1.1	1.2	1.3	1.4	1.5
$f(x)$	3	25	26	27	49	52	62	63

(a) Using the table above, give an approximation of $f'(1)$.

One can use several different points to get an approximation of $f'(1)$. For example,

$$f'(1) \approx \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{27 - 26}{1.1 - 1} = 10.$$

(b) Would a left-hand or a right-hand sum give a lower estimate of $\int_1^{1.5} f(x)\,dx$? Why?

Since $f(x)$ is an increasing function, the left-hand sum will give a lower estimate of $\int_1^{1.5} f(x)\,dx$.

(c) Using the table above, give upper and lower estimates of $\int_1^{1.5} f(x)\,dx$.

As we determined in part (b), the left-hand sum gives a lower estimate of $\int_1^{1.5} f(x)\,dx$ and similarly the right-hand sum gives an over estimate.

$$\begin{aligned} LHS &= 0.1(26 + 27 + 49 + 52 + 62) \\ &= 21.6 \end{aligned}$$

$$\begin{aligned} RHS &= 0.1(27 + 49 + 52 + 62 + 63) \\ &= 25.3 \end{aligned}$$

So we have

$$21.6 \leq \int_1^{1.5} f(x)\,dx \leq 25.3.$$