

6. (10 points) It is estimated that the rate people will visit a new theme park is given as

$$r(t) = \frac{A}{1 + Be^{-0.5t}}$$

where A and B are both constants and $r(t)$ is measured in people/day, and $t = 0$ corresponds to opening day.

(a) Write an integral that gives the total number of people visiting the park in the first year it is open. Do not try to evaluate the integral!

The total number of people visiting the park in the first year is $\int_0^{365} r(t) dt$.

(b) Suppose that $A = 100$ and $B = 5$. Given that

$$\frac{d}{dt} (2A \ln(1 + Be^{-0.5t}) - 2A \ln(Be^{-0.5t})) = \frac{A}{1 + Be^{-0.5t}},$$

use the First Fundamental Theorem of Calculus to evaluate how many people visit the park during the first year it is open. Make sure you clearly indicate your use of the theorem.

Let $R(t) = 200 \ln(1 + 5e^{-0.5t}) - 200 \ln(5e^{-0.5t})$. Then $R'(t) = r(t)$, so the First Fundamental Theorem of Calculus says that

$$\begin{aligned} \int_0^{365} r(t) dt &= R(365) - R(0) \\ &= 36,141. \end{aligned}$$

So 36,141 people visit the theme park during its first year.