7. (17 points) At the Wizard Fair, there is a booth where wizards win Bertie Bott's Every Flavor Beans. To determine how many beans one gets, a contestant is given a string 50 inches long. From this string, contestants can cut lengths to form an equilateral triangle and a rectangle whose length is twice its width. The number of Bertie Bott's beans one wins depends on the combined areas of the triangle and rectangle. Harry, knowing calculus, goes immediately to work setting up a function, finding critical points, etc.
(a) Use your knowledge of calculus to determine the areas of the triangle and rectangle that will maximize the number of beans that Harry can win. Show your work.
(b) If the number of beans won is 9 times the combined area, what is the greatest number of beans a contestant can win?
