6. (4+4+4 points) Harry Potter, Ron, and Hermione decide to attend the Wizard Fair. The newest ride at the fair, called The Coil of Doom™, is a spin-off on bungee jumping. Riders are attached to a special bungee cord which oscillates up and down. The riders’ position above the ground, in feet, is given as a function of time, \( t \), in seconds, by \( y = y_0 \cos(\omega t) + C \), with \( y_0, \omega, \) and \( C \) constants.

(a) The riders board from a platform 15 feet above the ground, are pulled upward until, 6 seconds later, they reach a maximum height of 165 feet. In another 6 seconds, riders are back at the initial position. The cycle repeats for one minute, at which point the ride ends. Using this information, determine an explicit formula for \( y \). [Show all constants in \textit{exact} form.]

The riders start from a platform 15 feet above the ground and reach a maximum height of 165 ft. The midline is \( C = 90 \) and the amplitude must be \( \frac{165 - 15}{2} = 75 \) feet. Since the ride starts at the bottom, \( y_0 = -75 \). The period is the time it takes the riders to return to their original position. So, the period equals 12 seconds. Since \( \omega = \frac{2\pi}{\text{period}} \), \( \omega = \frac{\pi}{6} \). This means that \( y = -75 \cos(\frac{\pi}{6} t) + 90 \).

(b) Find formulas for the velocity and acceleration of the riders as a function of \( t \).

\[
v(t) = y' = 75(\frac{\pi}{6})\sin\left(\frac{\pi}{6} t\right)
\]

\[
a(t) = y'' = 75(\frac{\pi}{6})^2 \cos\left(\frac{\pi}{6} t\right)
\]

(c) Show that the function \( y \) satisfies the equation \( \frac{d^2y}{dt^2} + \omega^2 y = K \), where \( K \) is a constant. What is the value of \( K \)?

\[
\frac{d^2y}{dt^2} + \omega^2 y = a(t) + \omega^2 y \quad \text{and from part (a) we know} \quad \omega = \frac{\pi}{6}
\]

\[
= 75(\frac{\pi}{6})^2 \cos\left(\frac{\pi}{6} t\right) - (\frac{\pi}{6})^2(75)\cos\left(\frac{\pi}{6}\right) + 90
\]

\[
= 75(\frac{\pi}{6})^2 \cos\left(\frac{\pi}{6} t\right) - (\frac{\pi}{6})^2(75)\cos\left(\frac{\pi}{6}\right) + 90\left(\frac{\pi}{6}\right)^2
\]

\[
= \frac{90\pi^2}{36} = \frac{5\pi^2}{2}
\]

So \( K = \frac{5\pi^2}{2} \).