6. (4+4+4 points) Harry Potter, Ron, and Hermione decide to attend the Wizard Fair. The newest ride at the fair, called **The Coil of Doom**$^{\text{TM}}$, is a spin-off on bungee jumping. Riders are attached to a special bungee cord which oscillates up and down. The riders' position above the ground, in feet, is given as a function of time, t, in seconds, by $y = y_0\cos(\omega t) + C$, with y_0, ω, and C constants.

(a) The riders board from a platform 15 feet above the ground, are pulled upward until, 6 seconds later, they reach a maximum height of 165 feet. In another 6 seconds, riders are back at the initial position. The cycle repeats for one minute, at which point the ride ends. Using this information, determine an explicit formula for y. [Show all constants in *exact* form.]

The riders start from a platform 15 feet above the ground and reach a maximum height of 165 ft. The midline is $C = 90$ and the amplitude must be $\dfrac{165-15}{2} = 75$ feet. Since the ride starts at the bottom, , $y_0 = -75$ The period is the time it takes the riders to return to their original position. So, the period equals 12 seconds. Since $\omega = \dfrac{2\pi}{\text{period}}$, $\omega = \dfrac{\pi}{6}$. This means that $y = -75\cos(\dfrac{\pi}{6}t) + 90$.

(b) Find formulas for the velocity and acceleration of the riders as a function of t.

$$v(t) = y' = 75(\frac{\pi}{6})\sin(\frac{\pi}{6}t)$$

$$a(t) = y'' = 75(\frac{\pi}{6})^2\cos(\frac{\pi}{6}t)$$

(c) Show that the function y satisfies the equation $\frac{d^2y}{dt^2} + \omega^2 y = K$, where K is a constant. What is the value of K?

$\dfrac{d^2y}{dt^2} + \omega^2 y = a(t) + \omega^2 y$ and from part (a) we know $\omega = \frac{\pi}{6}$

$$= 75(\frac{\pi}{6})^2\cos(\frac{\pi}{6}t) - (\frac{\pi}{6})^2((75)cos(\frac{\pi}{6}) + 90)$$

$$= 75(\frac{\pi}{6})^2\cos(\frac{\pi}{6}t) - (\frac{\pi}{6})^2(75)cos(\frac{\pi}{6}) + 90(\frac{\pi}{6})^2$$

$$= \frac{90\pi^2}{36} = \frac{5\pi^2}{2}$$

So $K = \dfrac{5\pi^2}{2}$.