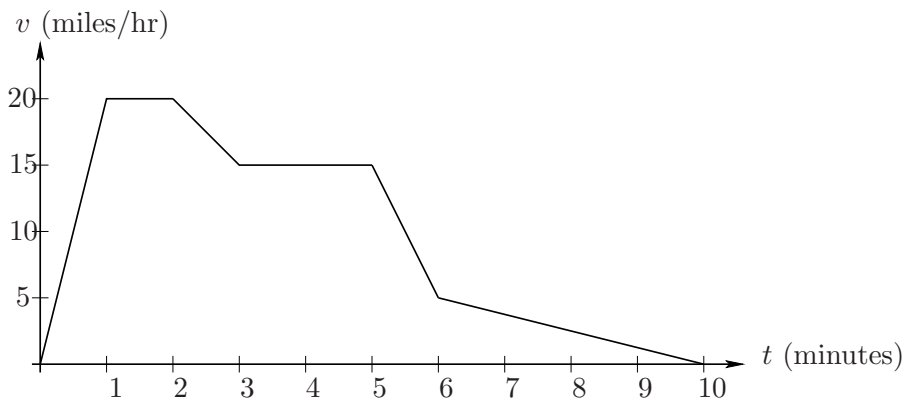


9. (5+2+2+3 points) The three happy wizards leave the fair and go home to watch the Simpsons. In this episode, Homer needs to deliver Lisa's homework to her at school, and he must do so before Principal Skinner arrives. Suppose Homer starts from the Simpson home in his car and travels with velocity given by the figure below. Suppose that Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner is able to sail through all the traffic and travels with constant velocity 10 miles per hour.



(a) How far does Homer travel during the 10 minutes shown in the graph?

To calculate how far Homer travels during the 10 minutes shown in the graph we find the area under the graph of Homer's velocity. Note that we must multiply by a constant so that the units are correct! This gives that distance $\frac{1}{60}(\text{area under curve}) = \frac{1}{60}(97.5) = 1.625$ miles.

(b) What is the average of Homer's velocity during the 10 minute drive?

$$\text{average} = \left(\frac{1}{10}\right) \int_0^{10} v(t) dt = 9.75 \text{ miles/hr.}$$

(c) At what time, $t > 0$, is Homer the greatest distance ahead of Principal Skinner?

As long as Homer's velocity is greater than Principal Skinner's velocity, Homer is becoming farther away from Principal Skinner. Since Principal Skinner is traveling at a constant velocity of 10 miles/hr, Homer is the greatest distance ahead of Skinner at $t \approx 5.5$ minutes.

(d) Does Principal Skinner overtake Homer, and if so, when? Explain.

Principal Skinner will overtake Homer when the distance he has traveled is equal to the distance that Homer has traveled. Notice though that the area under Homer's velocity curve and the area under Principal Skinner's velocity curve overlap. So, they will have traveled the same distance when the area between Homer's velocity curve and Skinner's velocity curve from $t = 0$ to $t = 5.5$ equals the area between the two velocity curves from $t = 5.5$ to some time $t > 5.5$. Notice though that the area between the two curves from $t = 0$ to $t = 5.5$ is greater than the area between the two curves from $t = 5.5$ to $t = 10$. So, Skinner does not overtake Homer. Or, more precisely, Principal Skinner travels $\frac{1}{60}(10)(8) = 1.33$ miles. This is less than the 1.625 miles that Homer traveled in the 10 minutes.