

1. (14 points) Problems (a), (b) and (c) below are independent of each other.

(a) (5 pts.) Compute the linear approximation to $g(x) = 3 \ln(x^2)$ near $x = 1$.

- $g'(x) = 3 \frac{1}{x^2}(2x) = \frac{6}{x}$;
- $g'(1) = 6$;
- $g(1) = 3 \ln(1) = 0$.

So, $g(x) \simeq 6(x - 1)$ near $x = 1$.

(b) (3 pts.) Write the limit definition of the derivative of the function $f(x) = e^x - e^{-x}$ at the point $x = a$. You do *not* need to simplify or attempt to compute the limit.

$$f'(a) = \lim_{h \rightarrow 0} \frac{e^{a+h} - e^{-(a+h)} - e^a + e^{-a}}{h}.$$

(c) (6 pts.) Assuming the following table accurately represents the behavior of the continuous function $s(x)$ over the interval $[0, 12]$, approximate the following:

[NOTE: the values in the table are for $s'(x)$, not $s(x)$].

x	0	3	6	10	12
$s'(x)$	-6	-3	0	1.2	17

(i) $s''(3) \simeq \frac{1}{2} \left(\frac{0+3}{3} + \frac{-3+6}{3} \right) = (1+1)/2 = 1$

(ii) All intervals in $[0, 12]$ (if any) over which s is decreasing.

If s is decreasing, then $s' < 0$. So, s is decreasing over $(0, 6)$.

(iii) All intervals in $[0, 12]$ (if any) over which s is concave down.

If s is concave down, then s' is decreasing. Since s' is increasing over all of $[0, 12]$, there are *NO intervals* over which s is concave down.