4. (12 points) Problems (a) and (b) below are independent of each other.

(a) (6 pts.) Suppose the function r gives the number of customers per day going to a new ice-cream store that just opened near campus. (Assume t is measured in days since the opening and that we are modeling the situation by a continuous function, r.) *IMPORTANT: The answers to (i) and (ii) should include clear units, and should be given using words understandable to someone who has never taken calculus.*

(i) What does $\int_0^{20} r(t)\,dt$ represent?

This definite integral represents the total number of *customers* in the ice-cream store during the first 20 days following its opening.

(ii) If each customer spends on average of \$3.50 in the store, what does the following expression represent?

$$\frac{3.5}{20}\int_0^{20} r(t)\,dt$$

This definite integral represents the *average daily revenue* (in *dollars*) of the ice-cream store (from sales to customers) during the first 20 days following its opening.

(b) (6 points) If the average value of the function $d(x) = 7/x^2$ on the interval $[1, c]$ is equal to 1, what is the value of c?

Using the definition of average value we see that:

$$1 = \frac{1}{c-1}\int_1^c \frac{7}{x^2}\,dx, \quad \text{which means that}$$

$$1 = \frac{7}{c-1}\left(\frac{-1}{x}\right)\Big|_1^c.$$

Now, the previous equation amounts to:

$$\frac{c-1}{7} = \frac{(1-c)}{c}, \quad \text{or}$$

$$\begin{aligned} c^2 - c &= 7 + 7c, \\ 0 &= c^2 - 8c + 7, \\ 0 &= (c-1)(c-7). \end{aligned}$$

Thus, either $c = 1$ or $c = 7$. Note that $\int_1^1 f(x)d(x) = 0$ for any function, so $c = 7$.