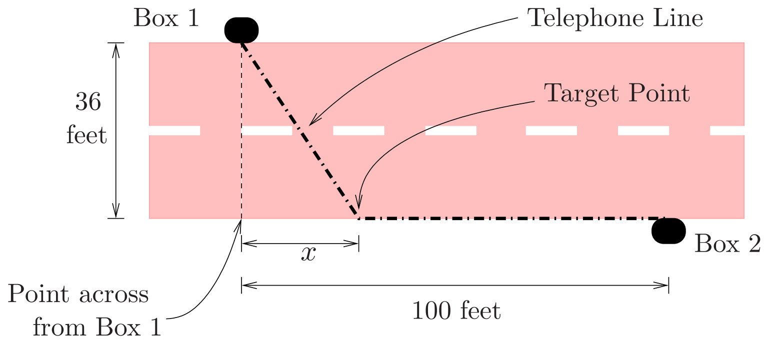
6. (12 points) A telephone installation crew must run a line underground between two junction boxes. Unfortunately, there is a 36 feet wide paved road between the two boxes, and one box is 100 feet down that lane from the other (see figure). It costs \$30 per foot to cut and repair the paved road, but only \$24 per foot to dig and refill along the side of the road. The crew will cut and repair the road to a point x feet from the point directly across from the first junction box, and then dig along the road the rest of the way. Determine the number of feet, x, from the point directly across from the first junction box which will minimize the cost of the installation.



Let C stand for the total cost of the installation. Note that C is a function of x. Then:

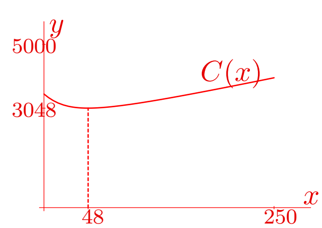
$$\begin{aligned} C(x) &= (\text{Total cost on paved road}) + (\text{Total cost along the road}) \\ &= 30\ (\text{Distance on paved road}) + 24\ (\text{Distance along the road}) \\ &= 30\sqrt{x^2+36^2} + 24(100-x) \\ &= 30\sqrt{x^2+1296} - 24x + 2400, \quad \textit{and} \quad C'(x) = \frac{30x - 24\sqrt{x^2+1296}}{\sqrt{x^2+1296}}. \end{aligned}$$

If $C'(x) = 0$, then:

$$30x - 24\sqrt{x^2+1296} = 0, \quad \text{which means that}$$
$$\sqrt{x^2+1296} = \frac{5x}{4}, \quad \text{or}$$
$$x^2 + 1296 = \frac{25}{16}x^2, \quad \text{or} \quad x = 48.$$

So the only candidate value for a minimum of $C(x)$ between 0 and 100, is $x = 48$ feet.

Looking (for instance) at the graph of C against x,



we see that $x = 48$ *is* a local minimum for C in the interval $[0, 100]$. [Note: we could have also used the first or second derivative test to show that $x = 48$ is a local minimum and then evaluated $C(x)$ at $x = 0, x = 48$, and $x = 100$ to show that the minimum occurs at $x = 48$–or, provided an argument that $x = 48$ is the only critical point on the domain of the function and then shown that the minimum occurs there.]

Thus, the crew should dig diagonally across the road to a point 48 feet from the point directly across from the first junction box in order to minimize the cost of the installation.