7. (12 points) You are heading due North across the Mackinac Bridge, and you sight your favorite fudge factory, located 1/4 mile due East of the end of the bridge. You might be fooling yourself, but the minute you sight the factory you are sure you can smell the chocolate fudge. You have to put the car on cruise control (at 55 mph) to resist the temptation to speed the rest of the way across the bridge. Show all work on parts (a) and (b) below.

(a) If you are still 1.25 miles from the end of the bridge when you spot the factory, what is the distance (across the water) between your car and the factory?

- $D = \sqrt{(1.25)^2 + (0.25)^2} \simeq 1.275$ miles.

So, the distance across the water between my car and the factory is about 1.275 miles.

(b) If θ is the angle formed by a line between the factory and end of the bridge and the line from the factory to your car, how fast is θ changing at the time you spot the factory?

- $\tan \theta = \dfrac{x}{1/4} = 4x$, where

- x is the distance between your car and the end of the bridge, and $\dfrac{dx}{dt} = -55$ mph, so

- $\theta = \arctan(4x)$.

Therefore,

- $\dfrac{d\theta}{dt} = \dfrac{4}{1 + (4x)^2} \dfrac{dx}{dt} = \dfrac{4(-55)}{1 + 16x^2}$, so

- when $x = 1.25$, we have $\dfrac{d\theta}{dt} = \dfrac{-220}{26} = -8.46$ radians/hr.

Thus, the angle θ is decreasing by about 8.46 radians per hour when you first spot the fudge factory!