3. (12 points) Octavius, the giant octopus, escaped after all. He was being kept in a temporary tank near the harbor—apparently even less secure than his tank at the zoo. He managed to get into the bay, but the coast guard could keep track of him with a homing device that had been attached to Octavius at the zoo.

(a) The coast guard station is 2 kilometers (2000 meters) down the beach from where Octavius entered the bay. If the octopus was moving directly away from the shore at a constant rate of 25 meters per minute, how fast was the distance between the coast guard and Octavius changing when the octopus was 200 meters from the shore?

Let the distance from Octavius to the shore be given by x, and the distance from Octavius to the Coast Guard station be given by D. Then $D^2 = x^2 + (2000)^2$, and

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt}.$$

We are given that $\frac{dx}{dt} = 25$ m/min, and when $x = 200$ m., $D = \sqrt{200^2 + 2000^2} \approx 2009.98$ m. Thus, $\frac{dD}{dt} \approx 2.49$ m/min when $x = 200$.

(b) At what rate is the angle formed by the beach and the line that gives distance from the coast guard station to Octavius changing when Octavius is 200 meters from the shore?

We know that

$$\tan\theta = \frac{x}{2000},$$

so

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = \frac{1}{2000}\left(\frac{dx}{dt}\right).$$

When $x = 200, \cos\theta = \frac{2000}{2010}$.

Thus,

$$\frac{d\theta}{dt} = \frac{(\frac{2000}{2010})^2}{2000}(25) \approx 0.01238 \quad \text{rad/min.}$$