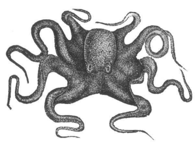
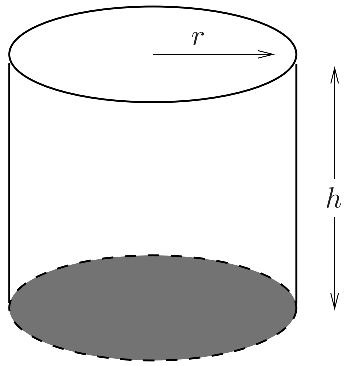
4. (12 points) The zoo has decided to make the new octopus tank spectacular. It will be cylindrical with a round base and top. The sides will be made of Plexiglas which costs \$65.00 per square meter, and the materials for the top and bottom of the tank cost \$50.00 per square meter. If the tank must hold 45 cubic meters of water, what dimensions will minimize the cost, and what is the minimum cost?





Let V denote the volume of the tank. Then $V = \pi r^2 h = 45\ m^3$.

Solving for h gives $h = \dfrac{45}{\pi r^2}$ m.

The area of the "sides" of the tank is $2\pi r h = \dfrac{90}{r}\ m^2$, and the area of the circles for the top and bottom of the tank is $2\pi r^2$.

Thus, the cost of the tank as a function of r is

$$C(r) = 65(\frac{90}{r}) + 50(2)(\pi r^2) = \frac{5850}{r} + 100\pi r^2.$$

To minimize the cost, set $C'(r) = -\dfrac{5850}{r^2} + 200\pi r$ equal to zero, since $C'(r)$ is defined for all values of r in the domain $(r > 0)$.

Solving for r, we get $r^3 = \dfrac{5850}{200\pi}$, so $r \approx 2.104$ meters.

Testing to see if this r value is in fact, the minimum, we use the second derivative test.

Since $C''(r) = 200\pi + 2\dfrac{5850}{r^3} > 0$ for all values of r in the domain, we see that C is concave up for all values of r. Since $C(r) \to \infty$ as $r \to 0$ and as $r \to \infty$, $r = 2.104$ is the global minimum of C.

radius ≈ 2.104 m

height ≈ 3.237 m

cost $\approx \$4171$