5. (16 points) Use the information given in the table below to calculate the indicated values. If a value cannot be determined, state explicitly what is missing. Assume that \( f \) and \( f' \) are continuous, and that the table is reflective of the behavior of \( f \).

\[
\begin{array}{c|ccccc}
 x & 0 & 3 & 6 & 9 & 12 \\
 \hline
 f(x) & 30 & 20 & 13 & 8 & 5 \\
 f'(x) & -4 & -3 & -2 & -1.5 & -.5 \\
\end{array}
\]

Determine the following and show your work (3 points each):

(a) an approximate value for \( f(3.1) \) using a local linearization

\[
f(3.1) \approx f(3) + f'(3)(3.1 - 3) \approx 20 + (-3)(0.1) = 19.7
\]

(b) a left-hand sum with 4 subdivisions to approximate \( \int_0^{12} f(x)dx \)

\[
\text{LHS}_4 = (f(0) + f(3) + f(6) + f(9))(3) = 213
\]

(c) the least number of subdivisions necessary to assure that the left- and right-hand approximations of \( \int_0^{12} f(x)dx \) agree to within 1 unit

If \( |\text{RHS} - \text{LHS}| \leq 1 \), then \( |f(12) - f(0)|\Delta x = 25\Delta x \leq 1 \). Thus, \( 25 \left( \frac{12 - 0}{n} \right) \leq 1 \Rightarrow (25)(12) \leq n \). This implies we need at least 300 subdivisions.

(d) \( \int_3^{12} f'(x)dx \)

From the FTofC, we know \( \int_3^{12} f'(x)dx = f(12) - f(3) = 5 - 20 = -15 \)

Explain your answers to the following (2 points each):

(e) Do you expect your approximation for \( f(3.1) \) from part (a) to be an overestimate or an underestimate?

If the table is representative of the behavior of the function \( f \), then \( f''(3) > 0 \) which implies that \( f \) is concave up at 3. Thus we expect the approximation to be an underestimate.

(f) Do you expect your left-hand approximation from part (b) to be an overestimate or an underestimate?

If the table is representative of the behavior of \( f \), then \( f \) is decreasing, thus the left-hand sum is an overestimate.