5. (16 points) Use the information given in the table below to calculate the indicated values. If a value cannot be determined, state explicitly what is missing. Assume that f and f' are continuous, and that the table is reflective of the behavior of f.

x	0	3	6	9	12
$f(x)$	30	20	13	8	5
$f'(x)$	-4	-3	-2	-1.5	-.5

Determine the following and show your work (3 points each):

(a) an approximate value for $f(3.1)$ using a local linearization

$f(3.1) \approx f(3) + f'(3)(3.1-3) \approx 20 + (-3)(0.1) = 19.7$

(b) a left-hand sum with 4 subdivisions to approximate $\int_0^{12} f(x)dx$

$\text{LHS}_{(4)} = (f(0) + f(3) + f(6) + f(9))(3) = 213$

(c) the least number of subdivisions necessary to assure that the left- and right-hand approximations of $\int_0^{12} f(x)dx$ agree to within 1 unit

If $|RHS - LHS| \leq 1$, then $|f(12) - f(0)|\Delta x = 25\Delta x \leq 1$. Thus, $25\left(\frac{12-0}{n}\right) \leq 1 \Rightarrow (25)(12) \leq n$. This implies we need at least 300 subdivisions.

(d) $\int_3^{12} f'(x)dx$

From the FTofC, we know $\int_3^{12} f'(x)dx = f(12) - f(3) = 5 - 20 = -15$

Explain your answers to the following (2 points each):

(e) Do you expect your approximation for $f(3.1)$ from part (a) to be an overestimate or an underestimate?

If the table is representative of the behavior of the function f, then $f''(3) > 0$ which imples that f is concave up at 3. Thus we expect the approximation to be an underestimate.

(f) Do you expect your left-hand approximation from part (b) to be an overestimate or an underestimate?

If the table is representative of the behavior of f, then f is decreasing, thus the left-hand sum is an overestimate.