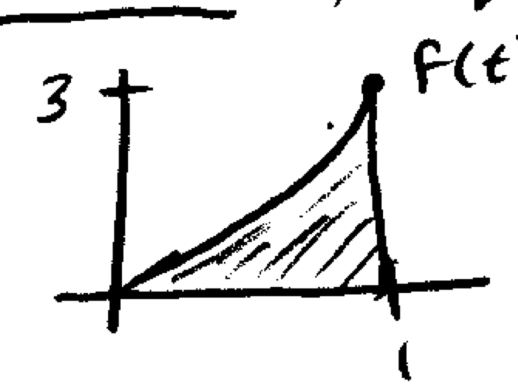
1. (2 points each) For each of the following, circle all the statements which are **always** true. For the cases below, one statement may be true, **or** both **or** neither of the statements may be true.

(a) Let $f(t) = t^2 + 2t$.

- $\dfrac{d}{dt}\displaystyle\int_0^1 f(t)\,dt = t^2 + 2t.$
- $\dfrac{d}{dt}\displaystyle\int_0^1 f(t)\,dt = 0.$

$\int_0^1 f(t)\,dt$ is a constant, equal to this area:



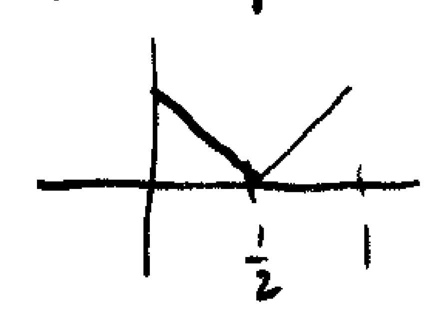
(b) Let $g(x)$ be continuous on the interval $[0,1]$.

- The limit $\displaystyle\lim_{n\to\infty}\left(\sum_{k=1}^{n} g(x_k)\cdot\frac{1}{n}\right)$ exists.

this is $\int_0^1 g(x)dx$, which always exists

- The limit $\displaystyle\lim_{h\to 0}\frac{g(0.5+h)-g(0.5)}{h}$ exists.

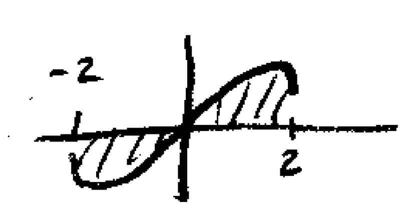
This is the derivative $g'(.5)$. It might exist, but might not, e.g. $g(x) = |x - \frac{1}{2}|$.



(c) Suppose $\displaystyle\int_{-2}^{2} F(x)\,dx = 5.$

- F is not an odd function.

if it were, the integral would be 0



- For some c in the interval $[-2,2]$, $F(c) > 1$.

if $F(c)$ were ≤ 1 on the whole interval, then we would have $\int_{-2}^{2} F(t) \le \int_{-2}^{2} 1\,dt = 4$.

(d) Suppose h is a differentiable function defined on $[a,b]$, with antiderivative H. Assume $h(t) > 0$ for all t in $[a,b]$

- h has either a local maximum or minimum (or both) on (a,b).
- H has a local maximum at b.

$H' = h > 0$ means H is increasing, so its max is at b.