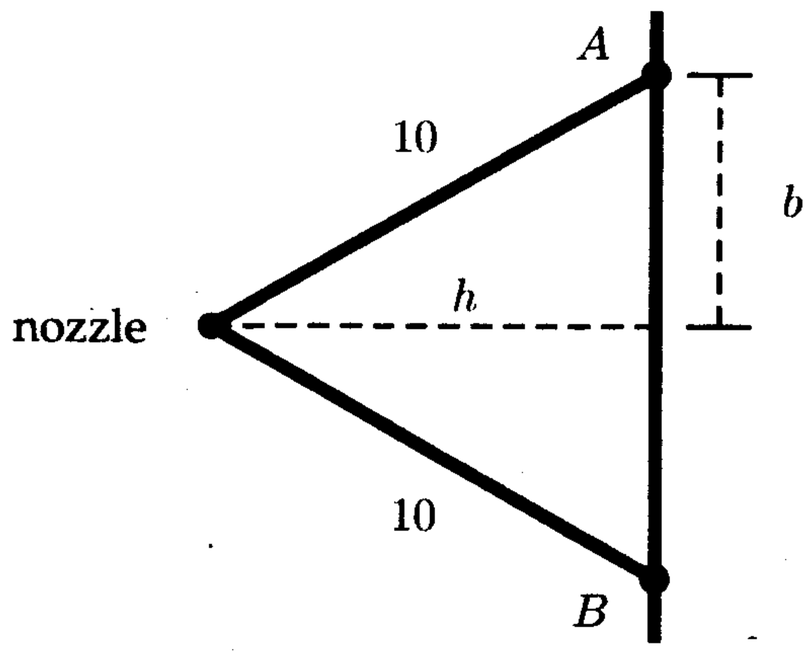
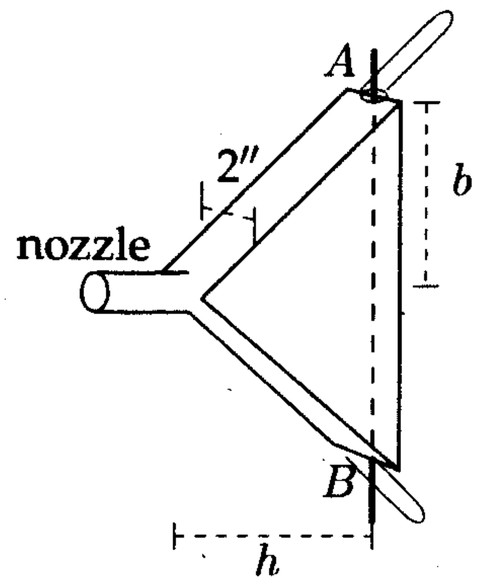
W08 RR

6. (10 points) A bellows[1] has a triangular frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below (the figure to the right shows a 3D sketch of the bellows; the figure to the left, a 2D sketch that may be specifically useful for solving the problem).





Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangular cross-section shown above times the height of 2. Suppose you pump the bellows by moving A downward towards the center at a constant speed of 3 in/s. (So B also moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)

We have:

$$b^2+h^2=10^2$$

$$V = \text{Vol} = 2\cdot\tfrac{1}{2}(2b)h = 2bh$$

We want:

$\frac{dV}{dt}$ when $\frac{db}{dt} = -3$ in/sec,

$b = 6$ in

Eliminate h:

$$h=\sqrt{100-b^2}$$

So

$$V = 2b\sqrt{100-b^2}$$

$$\Rightarrow \frac{dV}{dt} = 2\left[b\cdot\tfrac{1}{2}(100-b^2)^{-1/2}(-2b) + (1)\cdot(100-b^2)^{1/2}\right]\cdot\frac{db}{dt}$$

Now plug in:

$$\frac{dV}{dt} = 2\left[6\cdot\tfrac{1}{2}(100-6^2)^{-1/2}(-2\cdot6) + (100-6^2)^{1/2}\right](-3)$$

$$= -6\left[-36\cdot64^{-1/2}+64^{1/2}\right]$$

$$= -6\left[\frac{-36}{8}+8\right] = \boxed{-21 \text{ in}^3/\text{sec}}.$$

[1] A bellows is a device with a nozzle attached to a chamber; it is used to blow air out through the nozzle by reducing the volume of the chamber. In the bellows described here this is accomplished by moving the points A and B as indicated.