6. (10 points) A bellows\(^1\) has a triangular frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points \(A\) and \(B\) which can slide, as shown in the diagrams below (the figure to the right shows a 3D sketch of the bellows; the figure to the left, a 2D sketch that may be specifically useful for solving the problem).

\[
\begin{align*}
  b^2 + h^2 &= 10^2 \\
  V &= \text{Vol} = 2 \cdot \frac{1}{2} (2b)h = 2bh
\end{align*}
\]

We want:
\[
\frac{dV}{dt} \quad \text{when} \quad \frac{db}{dt} = -3 \text{ in/ sec},
\]

\[b = 6 \text{ in}\]

Eliminate \(h\):
\[
h = \sqrt{100 - b^2}
\]

So,
\[
V = 2b\sqrt{100 - b^2}
\]

\[
\Rightarrow \quad \frac{dV}{dt} = 2\left[b \cdot \frac{1}{2} (100-b^2)(-2b) + (1) \cdot (100-b^2)^{1/2}\right] \cdot \frac{db}{dt}
\]

Now plug in:
\[
\frac{dV}{dt} = 2\left[6 \cdot \frac{1}{2} (100-6^2)^{1/2}(-2 \cdot 6) + (100-6^2)^{1/2}(-3)\right]
\]

\[= -6\left[-36 \cdot 6^{-1/2} + 6^{-1/2}\right]
\]

\[= -6\left[-\frac{36}{8} + 8\right] = -21 \text{ in}^3/\text{sec}.
\]

\(^1\)A bellows is a device with a nozzle attached to a chamber; it is used to blow air out through the nozzle by reducing the volume of the chamber. In the bellows described here this is accomplished by moving the points \(A\) and \(B\) as indicated.