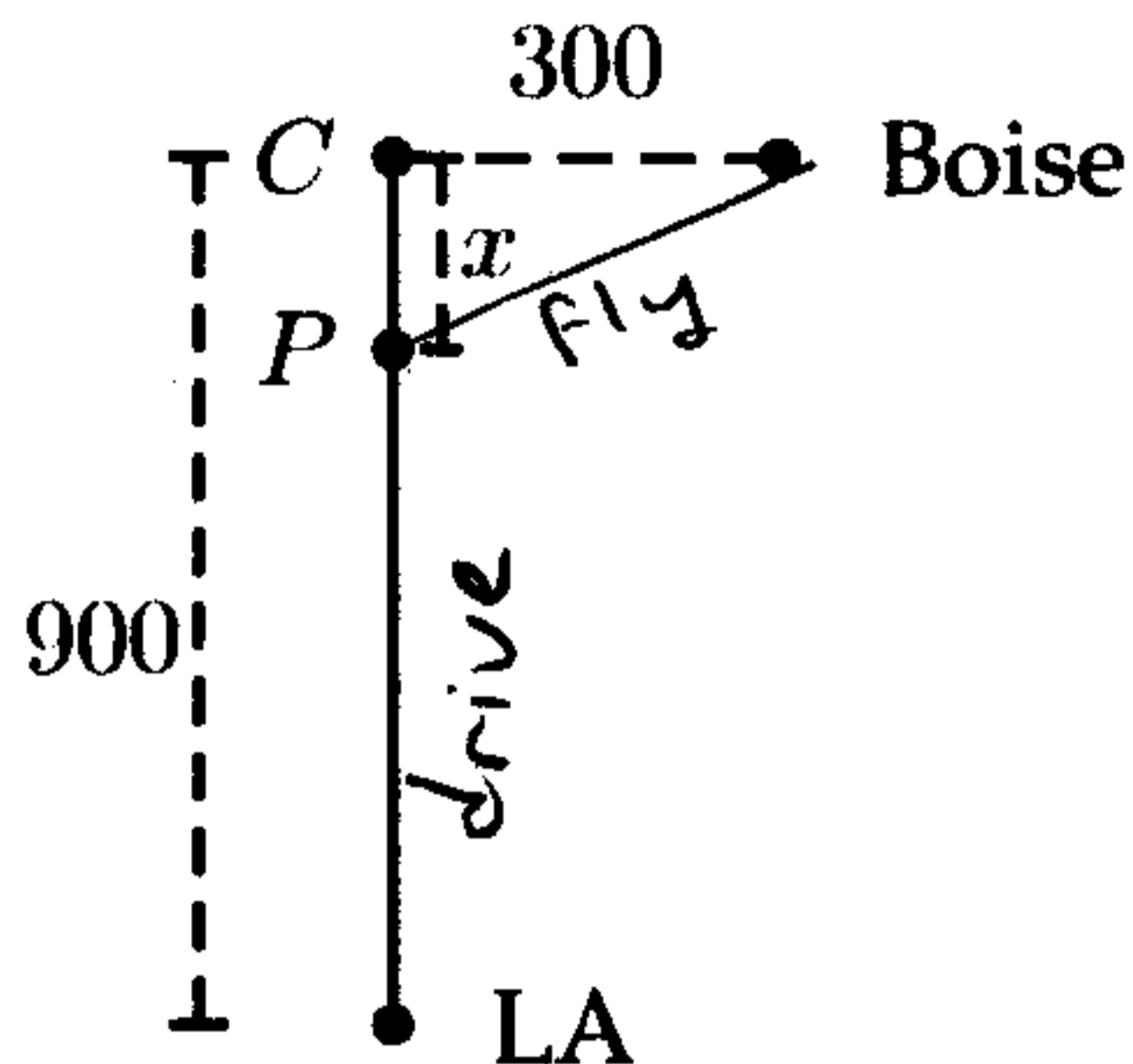


7. (12 points) You decide to take your two-week vacation from your job at the Idaho Potato Company in Boise to drive on California's coastal highway and visit friends in Los Angeles. Your plan is to fly a small plane to the coast, land at point P , and drive a rented car from there to LA. Assume the coast highway is straight, that C is the point on the coast directly west of Boise, and the distances (in miles) are as in the diagram below. (That is, ignore whatever actual experience you may have of that road!)

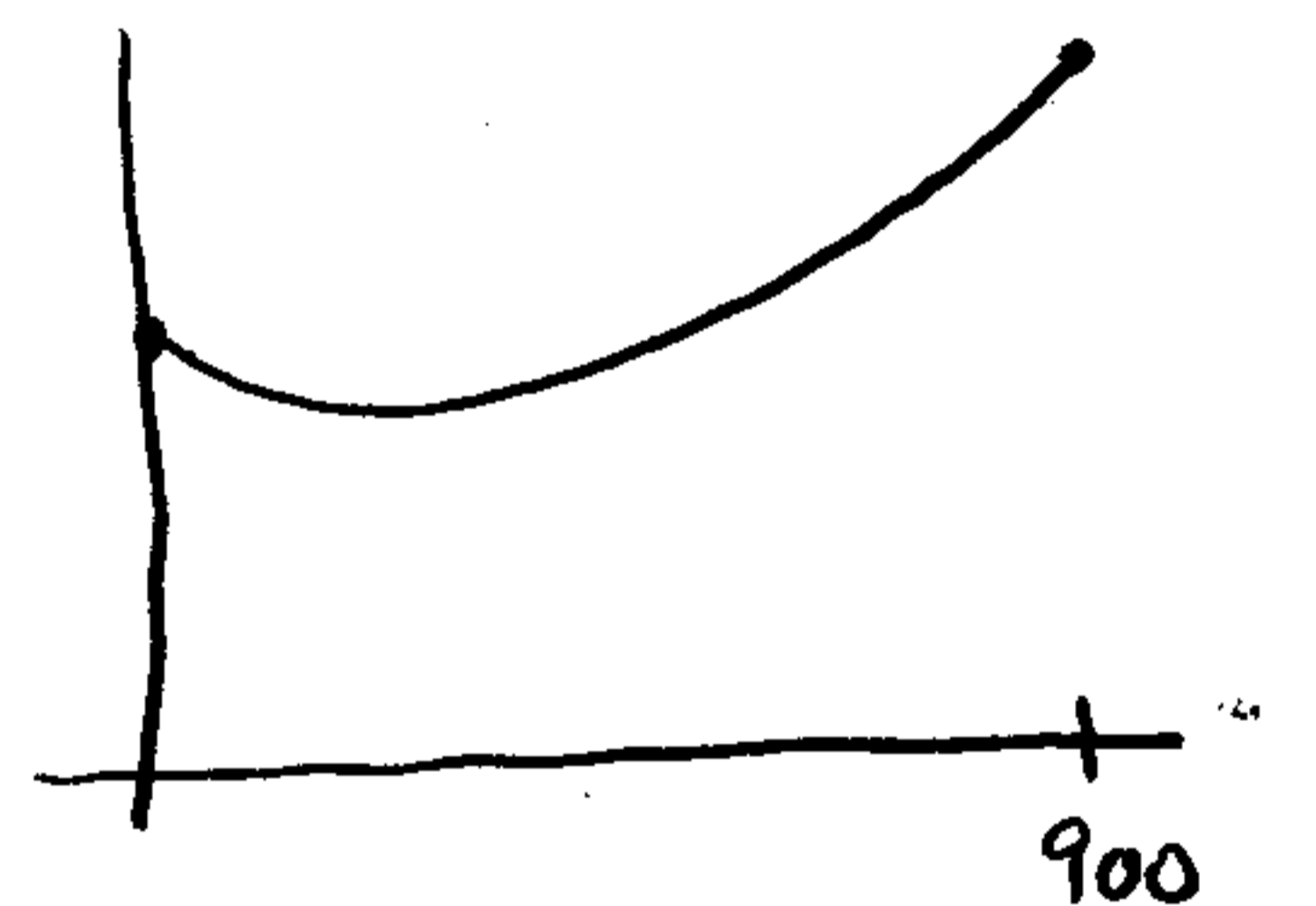


You want to minimize gas costs: it costs 30 cents per mile to fly, and 10 cents per mile to drive. Set up the function (of x) which describes the cost of gas for the trip, and find the position P where you should land to minimize the cost.

$$C = \text{total cost} = .30 \sqrt{300^2 + x^2} + .10(900 - x)$$

$$\begin{aligned} \frac{dC}{dx} &= .30 \cdot \frac{1}{2} (300^2 + x^2)^{-1/2} (2x) + .10(-1) \\ &= \frac{.30x}{\sqrt{300^2 + x^2}} - .10 \end{aligned}$$

Graph from Calc:



$$\text{So } \frac{dC}{dx} = 0 \Rightarrow \frac{.30x}{\sqrt{300^2 + x^2}} = .10 \Rightarrow 3x = \sqrt{300^2 + x^2}$$

$$\Rightarrow 9x^2 = 300^2 + x^2 \Rightarrow 8x^2 = 300^2$$

$\Rightarrow x = \pm \frac{300}{\sqrt{8}}$. It doesn't make sense for x to be negative (why fly north?), so

$$x = \frac{300}{\sqrt{8}} = 75\sqrt{2} \approx \boxed{106.7 \text{ miles}}$$

Check: Cost @ crit pt is \$174.85

Min must be at crit pt or at endpoints.

Endpoints: $x=0 \Rightarrow C = .30 \sqrt{300^2 + 0^2} + .10(900-0) = \180

$x=900 \Rightarrow C = .30 \sqrt{300^2 + 900^2} + 0 = \284.60

so crit pt is minimum.