3. The figure below shows a differentiable function $f$ and the line tangent to the graph at the point $(2, 5)$: (picture not drawn to scale)

(a) (3 points) Approximate $f(2.01)$. Is your approximation an over or underestimate? Explain.

From the two points given, the slope of the tangent line is 2. Therefore $f'(2) = 2$ and

$$f(2.01) \approx f(2) + 0.01 \cdot f'(2) = 5.02$$

Since $f$ is concave down this must be an overestimate.

(b) (3 points) Evaluate $h'(2)$ if $h(x) = (f(x))^3$.

By the chain rule,

$$h'(x) = 3\left(f(x)\right)^2 f'(x)$$

whence

$$h'(2) = 3\left(f(2)\right)^2 f'(2) = 150$$

(c) (3 points) Evaluate $g'(2)$ if $g(x) = e^{f(x)}$.

By the chain rule,

$$g'(x) = f'(x) e^{f(x)}$$

whence

$$g'(2) = f'(2) e^{f(2)} = 2 e^5 \approx 296.826$$